THE PLAN-NET AS A GEOMETRY FOR ANALYSIS OF PRE-MODERN ARCHITECTURAL DESIGN AND LAYOUT

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Limited Pre-Modern calculating skills favored geometry for design and layout. Technical limitations precluding durable, detailed measured drawings favored sequentially proportional steps transmitting information from design to construction. The Ancient Egyptian phrase “Casting the plan-net on the ground” implies a rectilinear network of geometrical lines serving to locate plan elements on the ground. Reconstruction of Polykleitos’ *Kanon* demonstrates design parameters based on sequential proportionality that extracts a “correctly” proportioned human figure from an original square base figure. Fifteenth century booklets describe extraction of a completed architectural form from the base figure. Iconographic sources trace these Ancient World methods from their use in practical implementation to symbolization as eighteenth century remembrances in Free Mason paraphernalia.

To associate floor plan elements with a rectilinear network, plan-net geometry manipulates proportional relationships of squares and rectangles in sequentially proportional steps. Geometrical design steps by divider and straightedge are identical to ground-lines steps by cord and peg, eliminating calculation from scale change.

Marking plan features by plan-net analysis reveals an inherent geometrical unity that appears to cross diachronic and synchronic borders. Varied plan-net patterns offer a new perspective for classifying vernacular floor plans. Conformance to plan-net lines by

indeterminate architectural elements validates elements in question and suggests other elements missing from the architectural or archeological record.

Seeking to understand how a house is thought as Henry Glassie said, and if culture is pattern in the mind, then plan-net analysis renders such pattern visible, to be understood as a unity crossing boundaries of culture and time but whose products can be differentiated as artifacts localized within cultural and temporal boundaries. To understand what has disappeared from the record, we must be willing to imagine what was, and then test what is imagined to ascertain how it fits to what is.

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Chapter I

INTRODUCTION

Geometrical analysis of architecture has long sought to explain building designs in terms of satisfying principles of harmony and beauty that were expressed in accord with the favored ideas of a local deity, royalty or upper class. Early analysts of historic architecture sought to show the presence of the Golden Section, the Fibonacci series, square root of two, the triangle, square, pentagon, hexagon or octagon as the governing element in the plan. Their analyses connected these figures to elements of philosophical, religious or folkloric significance rather than to demonstrating the actual steps by which the plan and ground lines were created.

Where literacy and modern calculating skills are lacking, design and control of building construction by systems that manipulate geometrical shapes rather than numbers requires processes to be thought through in a way that differs greatly from the modern practice based on numerical measurements from scaled drawings and blueprints, from which the builder transfers measurements to the material. The conceptual context for a methodology based on manipulation of geometrical shapes is for the most part lost in the distant past and must be reconstructed if we are to understand the processes. Thus the material for this study is of necessity drawn from all periods of history but it is unified in its intent by focus on its contribution to two core ideas, the usefulness of root rectangles generated from the diagonal of a square and the usefulness of proportionally constructed sequences.

The dissertation proposes an alternative way to think about the methodology of design and construction control that I will call the plan-net method. Henry Glassie sought "an account not of how a house is made, but of how a house is thought...."[1] Two issues regarding 'how a house is thought' are fundamental to the present work. The first is that systematic proportional geometry unifies in a single system both the procedure to lay-out a floor plan drawing and to draw the same plan as ground lines at the construction site. Remembering that Ground lines are no less a drawing than are the floor plan lines on paper it uses identical manipulative steps at the size of the drawn plan and at ground line size, thus eliminating calculation needed to make this transition arithmetically.

The second issue demonstrates that a unified system of sequentially proportional steps transmits from the last completed step the construction data that governs completion of the next step. This is so because the lines constituting each prior step are the dimensional template for elements of the next step. The plan-net process thus unifies in a single system both the sequence of developmental steps from beginning to end and the means of transition from small scale drawing to large scale ground lines. These considerations solve two fundamental problems of construction without measured drawings; that of scaling the work up to construction size and transmitting the relevant construction criteria to the work itself throughout the entire building process.

Glassie observed that "culture is pattern in the mind."[2] Plan-net geometry is a tool to transform pattern in a unified way from its potential state in the mind to its realized state as a material object. The structured thought process existing in the mind of the designer/builder constitutes a goodly part of the maker's competence. He noted that "the goal of analyzing for architectural competence is to create a systematic model that

[1] Glassie, *Folk Housing in Middle Virginia*, (Knoxville: The University of Virginia Press, 1975), 21
[2] Henry Glassie, *Folk Housing* 17.

accounts for the design ability of the idealized maker."[3] The process of plan-net methodology is just such a model because any successfully completed plan-net analysis ends with faithful reproduction of the floor plan under analysis and does so indistinguishably from the work of the idealized maker. Glassie said further that "focus on the structural rules to which the designer/builder is committed by virtue of the pattern he is following prevents a chaos of unorganized details in the finished work."[4] The unifying capacity of plan-net geometry mentioned above satisfies this goal. Plan-net analysis examines how a building is thought by examining a mental process with which the space in a building can be organized. Plan-net geometry a means that enables pattern in the mind to be realized as a material object. The ability to do this is a significant part of the architectural competence of the maker.

One may design and subsequently control the building process either by manipulating geometrically constructed shapes or by manipulating pure numbers. Two principle advantages of geometric manipulation are that a floor plan can be created without numerical calculation and without calculation a plan can also be changed from design size to ground-line size. The principle advantages of numerical calculation are that an infinitely divisible number system offers more precise control of size, shape and internal spatial relationship than geometrical construction does and that numerically recorded information on paper can be universally shared by any who can read numbers. Transition from control by geometrical measure to arithmetic measure marks a paradigm change in the methods by which the building process is organized.

[3] Glassie, *Folk Housing,* 17.
[4] Glassie, *Folk Housing.* 20.

At the core of Glassie's architectural grammar is his observation that "The plan of each house included a square." Plan-net analysis begins with identification of an initial square embedded in the lines of the floor plan, a square from which the overall plan will be generated. To complete the analysis a sequence of developmental steps follows, all of its steps proportionally related one to another. "All of the other dimensions of the house" he continues, "are determined by adding or subtracting units to or from the width of that square."[5] He says further that while the architectural grammar is laid out in an orderly sequence that appears to be chronological from rule set I to rule set IX, the individual designer/builder may "start at any point, take any route and arrive at the same end – the whole design of the house ...."[6] In a sequence of plan-net steps all subsequent steps add space. They never subtract space but they may divide any unit of space generated up to that point by halves and quarters.

Like Glassie's system, the plan-net process is chronologically directed because it is sequentially ordered. It emanates however, from an initial square rather than at any point in the process. It allows for freedom of choice regarding the next step to be taken but choice of a specific next step precludes development along certain other lines of possibility. The development process is organic because the characteristic by which it grows in size and complexity is that each step forward is generated from and constrained by data developed by the previous growth stage. These constraints derive from decisions regarding cultural and practical issues that affect the broad structure of the sequences that govern such determinations as building type, cultural associations and chronological periods.

[5] Glassie, *Folk Housing.* 22
[6] Glassie. *Folk Housing.* 36.

While measured drawings are common in print texts from the eighteenth century on, manipulation of geometric figures is barely hinted at in historical texts. For the most part the methods by which design and layout were practiced were taught through apprenticeship practices rarely written down or published. Procedures for geometrically manipulating space became explicit in publications only from the middle of the fifteenth century onward and then were not fully explained.

In an environment where apprenticeship is the norm for teaching and learning construction practices, the following characteristics imply a practical methodology very different from the modern norm; 1. Most people lack literacy and calculating skills. 2. Technological limitations severely restrict the size and portability of drawing surfaces available for measured and detailed large scale drawings. 3. Lacking portable measured drawings, control of the construction process proceeds by sequentially connected steps wherein the previous step lays out the information needed to take the following step. 4. Divider and straight edge on paper and cord and pegs on the ground enable non-calculated scaling change from drawing size to ground-line size by using the same geometric steps at both scales. 5. Design details are transmitted from the plan to the work material by the use of templates, an analogue rather than a digital process.

Transmitting design details to the work material at the job site by means of templates involves constructing a geometrical figure in either one, two or three dimensions from which at the work site the template dimensions can be copied directly to the multiplicity of worked pieces. A divider set to a specified length to copy an interval on a drawing from one place to another functions as a template. Likewise, a cord of specified length is a one dimensional template for cutting logs to the correct length to fit a specified position in a log house because it can be carried from log to log to mark off the dimension. These steps physically match the size or shape of one object to another.

The above mentioned characteristics imply a design and layout practice significantly different from the modern method. Plan-net analysis shows that these factors merge to unify both design and layout in a single geometric system, especially when the function of plan designer and construction supervisor come together in the single person of the master builder or the knowledgeable vernacular builder. The reliable vernacular builder, the architect and the master builder of cathedrals were guided by the same principles and procedures. What differed between the two levels of practice is the number and sophistication of design tools in the toolbox of each.

Today the term measurement is principally understood as the use of counting and measuring with a modular rule graduated by numbers. Many apprentice-based contexts understood measurement primarily in terms of a geometric construction to which numeration was secondarily applied for the purpose of quantification. To give an example specific to the construction trades, most floor plans of the Early Modern period in Germany gave total length and width of a building but lacked all detail measurements. Alongside the drawing there was often a scale divided into modular units, to which intervals taken from the drawing with the divider could be compared to determine a length expressed in number of units. Thus the plan was drawn without quantification, which was applied secondarily from the scale alongside the drawing to meet any need for numerical measurement when scaled up from drawing size to ground-line size.

Nevertheless, number as an important factor in design and layout cannot be totally dismissed since such measurement has always found a proper place in the dimensioning and placement of smaller elements, such as hearth size and position and size of doors and windows. Elements of conveniently small size could be measured with a ruler, a limitation that disappeared with the invention of the retractable tape measure in the decade of the 1850s. This invention combined the advantages of an infinitely divisible number system with that of the cord and peg method. Geometry played a determinative role in the processes of organizing space prior to the worldwide rise in the use of numbers, a concurrent increase in calculating skills and the invention of the tape measure. The change to a basis in numerical calculation occurred concurrent to a greatly increased interest in the use of numbers for recording data for the purpose of demonstrating empirically observed change in natural and cultural processes.[7]

John Harvey notes in *The Medieval Architect* that

> So far as there is direct surviving evidence for systems of medieval proportion, it indicates that they were the result of direct use of geometrical methods, and never involved calculation. Arithmetical and algebraic functions are there, but they went unrecognized by the craftsmen who reached their objective by a different road.[8]

Harvey's observation that 'arithmetical and algebraic functions are there, but they went unrecognized' suggests another question central to the dissertation. What elements of design and layout practiced at the level of monumental construction were recognized, shared by and used in the everyday vernacular world of house and barn construction? Certainly some of the complicated geometrical techniques of monumental construction were beyond the scope of most vernacular builders.[9] The boundaries of practice between

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[7] For a concise treatment of the spread of numeracy in early America, I recommend Patricia Cline Cohen, *A Calculating People*, (New York: Routledge, 1999).

[8] John Harvey, *The Medieval Architect*, (London: Wayland Publishers, 1972). 98.

[9] For the issue of literacy, calculating skills and Medieval masons, please see Lon R. Shelby, *Gothic Design Techniques*, (Carbondale: Southern Illinois University Press, 1977), and his numerous journal articles on the subject. See also John James, *Chartres, The Masons who Built a Legend.* (Boston: Routledge and Kegan Paul. 1982).

upper class and vernacular builders are difficult to determine because the term architect blurs with that of master builder and the terms become even more indistinct when one is referring simply to 'the man in charge of building this vernacular dwelling house.' What then were the simpler elements of geometry used in the palatial and monumental level of practice that were also practiced at the everyday vernacular level? Plan-net thinking will be seen to be among these less complex and esoteric elements used commonly by the vernacular builders. Though the complexity and sophistication of the steps the vernacular builder took might be much simpler, the work of a reliable builder was guided by the same kind of patterning activity as the upper class builder. In both cases the pattern had to be generated, either in the mind or on paper, and then laid out on the construction site as a very large scale drawing in the form of ground lines prior to construction. This world view was analogue, setting out one real world size or shape whose purpose was to be copied to another object to determine its size or shape rather than digital where shape is represented and transmitted as of a set of numbers. In figure one Matthes Roriczer's geometric base figure for extracting the elevation of a pinnacle was constructed by the manipulation of intervals and shapes. It will be seen in a later chapter how the elevation of the pinnacle is extracted from this base figure. These sizes and shapes are then transmitted by a template to the stone cutter's shop for the sizing and shaping of the necessary stone blocks. The significant point is that all of those determinations on the assembled blocks of stone considered as an object erected in three dimensions are set by this diagram. The present task is to identify and validate that the nature of plan-net geometry is a process sufficiently simple to be practical in a vernacular world largely lacking in literacy and calculating skills. That is to say, in a world dependent on real

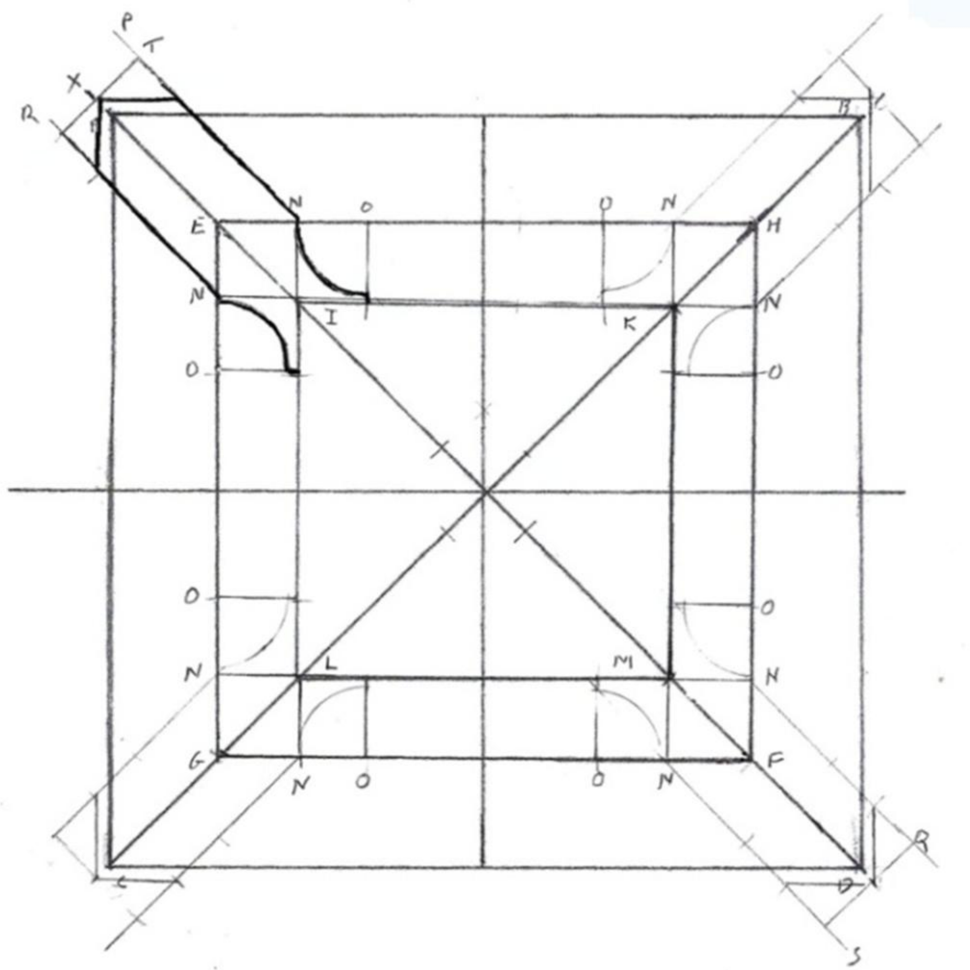
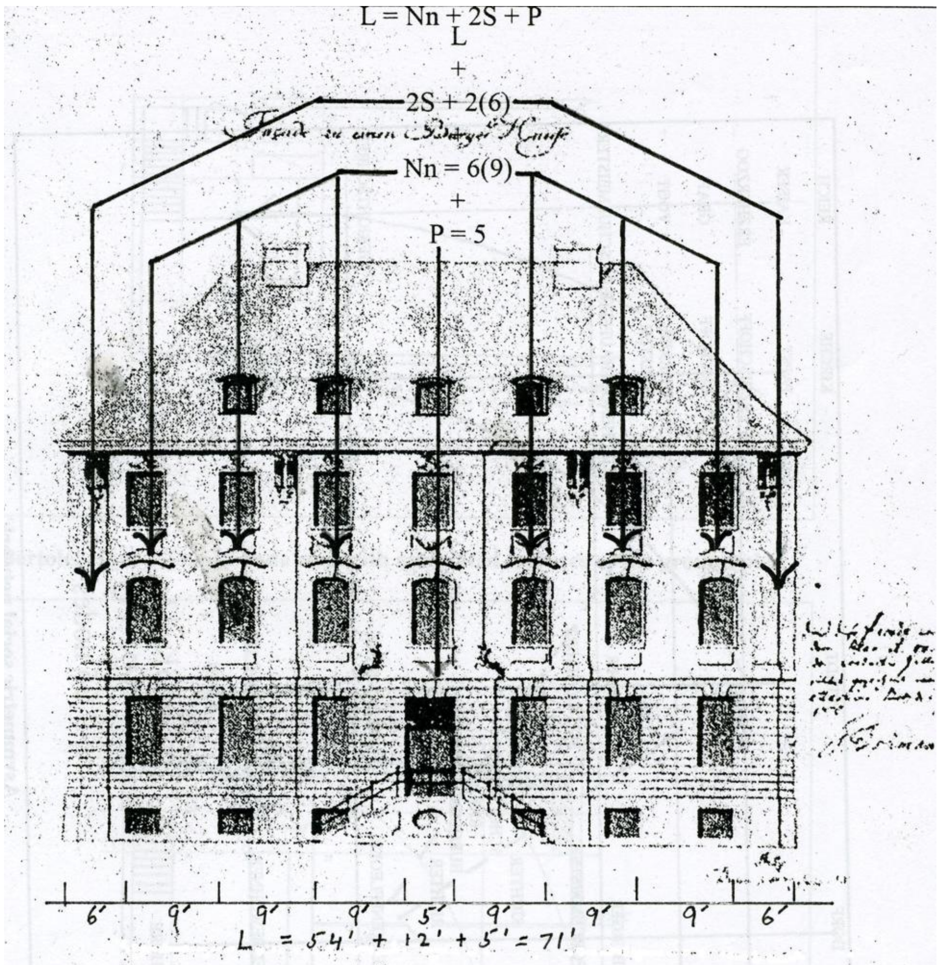


Fig. 1. Matthes Roriczer's base figure from which to extract the shape for a pinnacle, from *Büchlein der Fialen Gerechtigkeit, 1486.*

forms manipulated in real space; architectural problems solved by manipulation of intervals and shapes rather than numbers and transmitted from plan to construction by means of templates matching shape to shape.

By the latter part of the eighteenth century digital approaches to design were appearing in print. An emerging digital way of thinking about architectural planning and construction developed slowly and its level of development corresponded roughly to the extent to which the vernacular world also used arithmetic in commerce, trade and money



§. 501. Wer allgemeine Auflösungen liebt, kann sich auch hier leichter helfen. Es sey die Länge der Wand = L, die Vielheit der Fenster und Schäfte = n, die Breite eines Fensters und Schaftes = N, die Breite eines Schaftes = S und die Breite der Thüre = P; so ist für eine Wand mit

1. einer Thüre $L = Nn + 2S + P$, daher $L - 2S \mp P = Nn$; mithin $\frac{L - 2S \mp P}{N} = n$.

Fig. 2. (Below)The 1798 formula by Lorentz Suckow for a symmetrical Baroque facade. (Above) Applied by the author to a baroque facade drawing by the German architect David Gilly, d. 1808.

management. The advantage of digital problem solving lies in the fact that numbers are infinitely divisible and thus infinitely variable. A sufficient quantity of appropriately calculated numbers can define any shape imaginable to any desired degree of precision. [10] Numbers organized in a formula constitute the virtual representation of a real condition as indicted in the formula for a symmetrical Baroque facade by Lorentz Suckow illustrated above. Shapes demarcated by geometry are limited by the rules of geometric construction. Interest in change from analogue geometry to a digital basis became evident in print during the latter part of the eighteenth century as seen in figure two.

In Jena in 1798, Lorentz Johann Daniel Suckow published an architectural manual entitled *Erste Gründe der bürgerlichen Baukunst,* "The Principle Basis of Civil Architecture." In it he presented mathematical formulas to determine the placement of corner masonry columns, windows, doors and other details for a variety of balanced symmetrical facades for Baroque buildings. In figure two I have applied one of Suckow's formulas to the facade of a building designed by another German architect of the same period, David Gilly, who died in 1808. This was a transition from relying on the shapes and forms that geometry made available to crafting shape and organization precisely according to specific needs and desires. Design and implementation are no longer 'received' elements, but rather, "expressions of intent.'

Chapter two examines earlier academic studies of architecture's use of geometry in solving problems of shape and form to demonstrate the change in analytical goals from understanding the source of beauty in architecture to understanding the "how to" aspects. Early geometrical analysis of architecture focused on demonstrating the cause of harmony and beauty to be a geometrical figure inherent in the building floor plan, doing so by superimposing geometrical constructions on a floor plan and looking for coincident

[10] I am grateful to Dr. Hans-Helmut Görtz of Freinsheim, Germany for suggesting to me the use of the terms digital and analogue

points between diagram and plan. Only lately and gradually did analysis shift focus, seeking to understand the role of geometry relative to the how-to aspect of laying-out plan and ground lines. This change sets the stage to look now for evidence that something such as plan-net methodology existed.

Chapter three examines two core ideas that emerged from the transition of academic focus to practical rather than aesthetic considerations; the role of root rectangles in design and layout and also the role of sequential proportionality in transmitting information from the designer to the builder. At the foundational level of such understanding is the Ancient Greek concept of *dynamis* that empowers some elements of a system in transition to remain unchanged while other elements change. The proportional inter-relationships among the circle, triangle and square come together in manipulation of the square as the principle base figure for design and layout process in this study. The diagonal and side of the root rectangle are key elements in a generative system that extracts a completed figure from the initial square base figure in a process that is sequentially proportional. Finally, these elements come together in a fifteenth century publication that describes how circle, triangle and square inter-relationships are used to extract a completed architectural figure from an initial square base figure.

Contributing to our understanding of these issues are the *Meno* problem from Plato's dialogue of the same name, the *Kanon* of the ancient Greek sculptor Polykleitos, the Vedic Sulvasutras governing the construction of brick altars in India of the 7th century CE and Matthes Roriczer's two booklets on extracting the elevation for cathedral pinnacles and gablets from the initial square base figure.

What then, does constitute a plan-net methodology? Chapter four presents a straightforward and simple method for plan-net construction that extracts a plan-net from the initial square base figure by manipulating existing elements to form additional quadrilateral figures. These figures consist of rectilinear parallel lines whose intersections form nodes from which in turn subsequent steps extract further plan-net elements. If the initial base figure has been correctly sized and located, elements of the floor plan under analysis fall precisely on the lines and nodes of the plan-net. But what is the evidence for the concept of a plan-net, and the evidence that such a method lived on until problem-solving by geometrical manipulation was replaced by manipulation of numbers?

Hieroglyphic texts inscribed in Egyptian temple foundations introduce the idea of "casting the plan-net on the ground." The plan-net hypothesis is tested on an Egyptian *ostrakon* from the Valley of the Kings that contains a simple floor plan. The plan-net as a network of parallel rectilinear lines used in everyday practice is the most plausible explanation of the twelfth century Illumination of the Dream of Gonzo. The function of cord and peg geometry as the transmitting element from the ideal realm of the divine cosmos to the ground lines on the construction site is the implied narrative in the fifteenth century *Traditionskodex* of the monastery at Weissenau bei Konstanz. Lastly, the symbolization process of which the Weissenau document communicates a stage in its development, comes to completion in the constellation of ritual symbols of the eighteenth century Order of Free Masonry where the plan-net grid, the initial square base figure and the cord for "casting the Plan-net on the ground" stand as symbols in the progress of the Free Mason.

Chapter five applies the method of plan-net analysis to twenty four floor plans from a broad variety of cultural and chronological contexts. Because all analyses emanate from an initial square base figure, the analytical development can proceed in four directions from the initial square. Analyses that advance along a single axis are termed linear and those that advance along two perpendicular axes are termed rectilinear, though no analysis was found to advance along a single axis only. Analyses are classified according to the number of directions in which advancement occurs as quadrilateral rectilinear, trilateral rectilinear or bilateral rectilinear. The chapter applies plan-net analysis to a floor plan to determine which partitions are likely to be original and which are later additions, the thesis being that when all other elements do conform to plan-net lines and nodes, non-conformity indicates a non-original element. In the case of Palladio' Villa la Rotunda, by rendering the plan-net lines visible an implicit message is becomes evident. The chapter closes by showing that the vernacular use of plan-net lines constitutes a portion of the larger architectural context as a whole that vernacular builders readily understood and found directly useful at the level of their problems in building construction.

Chapter II

Previous Works

By the early nineteenth century numbers became the principal language for modeling real processes and things. With the advent of the retractable tape measure, the circular saw and other modern advances, the geometrical design and lay-out systems of an earlier age were eventually discarded and then forgotten. Analytical interest in geometry and architecture took the place of practical interest, geometry being then understood as a means to identify the basis for beauty in architecture. The principle methodology was to demonstrate that some favored geometrical figure or combination of figures was embedded in the building floor plan or elevation, bringing it into view by superimposing the favored geometrical diagram on the drawing and looking for coincident points between geometrical diagram and drawing. Circles, triangles squares, hexagons and octagons were the most common forms used.

Only relatively recently did analysis shift to the task of understanding the role of geometry in regard to the how-to aspect of laying out plan and ground lines and executing construction. By the mid-twentieth century, material culture studies took a practical interest in design by geometry seeking to understand how geometry might have controlled the erection of buildings. That change in direction provides the context in which we will look for evidence that something such as plan-net methodology existed. In the following material, the first three authors, Jay Hambidge, Kenneth Conant and Rudolf Helm represent studies seeking the source of harmony and beauty in architecture and the eight writers following focus on how geometry might have been used to lay out plans and ground lines and its role in governing the construction process.

Jay Hambidge and Dynamic Symmetry

In the first two decades of the twentieth century Jay Hambidge published on the concept of 'dynamic symmetry' whose origin he attributed to the *harpedonaptai,* the rope-stretchers of ancient Egypt.[11] Hambidge's application of the word dynamic to symmetry derives from the Ancient Greek concept of *dynamis,* the power at work in a system change that maintains stasis in some elements of the system while other elements change. He applied the term dynamic to a kind of symmetry that has the power to achieve an end or goal, specifically, to endow the finished work with an engaging vitality. Chapter three will examine *dynamis* and the *Kanon* of the Greek sculptor Polykleitos as a practical solution employing *dynamis*. Hambidge proposed that this is accomplished by a simple geometric method involving the diagonal of quadrilateral figures and he analyzed Greek temple plans and elevations in terms of this principle. Symmetry, he said, seeks "the just balance of variety in unity."[12] It is the relation that the part bears to the whole. He distinguished between static and dynamic symmetry. Static symmetry depends on the even modularity of its parts. He did not define dynamic symmetry in *The Elements of Dynamic Symmetry* but in the glossary of the Dover edition there is cited a notation from Heath's *Euclid,* Volume III, page 11, to the effect that Hambidge meant the Greek *dunamei summetros,* that I translate as the dynamis of symmetry, and Heath translated as 'commensurable in square.'

[11] Jay Hambidge, *The Parthenon and other Greek Temples*. *Their Dynamic Symmetry,* (New Haven: Yale University Press, 1924).

[12] Jay Hambidge, *The Elements of Dynamic Symmetry,* (New York: Dover Publications, Inc. 1967)., xii. Originally published by Brentano's Inc. in 1926. See also Hambidge, *Dynamic Symmetry in Composition as used by Artists,* (New York: Brentano's, 1923.)

Hambidge pointed out that this is a property of root rectangles.[13] Root rectangles are formed when the diagonal of the square rotated to one side forms a rectangle whose length is equal to the diagonal of the square, sides relating in length as $1 : \sqrt{2}$. By rotating the diagonals of a series of increasingly long rectangles this process can be extended as $1 : \sqrt{3}$, $1 : \sqrt{4}$, $1 : \sqrt{5}$ and so on. $\sqrt{3}$ and $\sqrt{5}$ calculate as incommensurable infinite repeating decimals, that is to say, irrational. The Greeks did not consider them so, because the values of their squares are commensurable.[14] Plan-net geometry is based on root rectangles created by swinging the diagonal of quadrilateral figures to produce a rectilinear network of parallel lines upon which a floor plan can be mapped.

Hambidge's analyses selected favored triangles as geometrical figures and super-imposed them on the floor plans to find the points at which they coincided with the floor plan lines. Upon finding sufficient points of coincidence, he argued that that specific triangle governed the lay-out of that floor plan. His analyses do not however, constitute a working method to produce a specific plan, but instead, sought to identify the source of harmony and beauty in the temple floor plans. Nevertheless his investigation of proportional relationships inherent in simple geometric figures, especially the root rectangle and the triangles created by its diagonals, forms an important part of the procedural basis for this dissertation.

Kenneth Conant and Design of Cluny III

Kenneth J. Conant published extensively between the 1920s and the 1970s on the principles by which the Abby Church at Cluny was designed and its ground lines laid out. Writing in an article on the continuation of Vitruvian tradition into the middle ages he

[13] Jay Hambidge, *The Parthenon*. 129.
[14] Hambidge. *The Parthenon*, 18.

argued in 1968 for such continuation as expressed in a variety of rectangular forms that can be generated by manipulation of the diagonal of the square.[15] In this article he identified six shapes to which he attributes Vitruvian influence, shapes given names by researchers attached to the National Museum at Ljubljana, Yugoslavia.[16] All are created from the square by swinging the diagonal of the square or a part of the square. These shapes are the *square* with sides related as 1 : 1 and found in four churches including Cluny III, the *diagon* (1 : √2) found in four churches including Cluny III, the *hemiolion* (2 : 3) found in two including Cluny III, the *auron* (1 : 1.618, or f) found in 3 churches including Cluny III, the *dual diagon* { 1 : 2(√2-1)} found at Cluny III and finally, *embracing diagons*, a special case of the double square, found also at Cluny III. He suggested that because the diagon is found so frequently in the ancient world, the Romans did not limit it to the Vitruvian atria of the third class. The principle of the diagon being the relation of the length of one side to the length of the diagonal, it can be extended to the series, 1 : √2, 1 : √3, 1 : √4, 1 ; √5 and so on simply by swinging the diagonal of each increasingly long rectangle. Conant reports that the Ljubljana program recognized individual houses at Emona that had the measurements of the auron and the diagon, with discrepancies that range from 1 to 2 per cent.

He noted also that Vitruvius says plans were worked out with the rule and compass, or on the ground with cord and pegs. Without textual support he observes that the diagonal "is available to determine supplementary dimensions." This observation is directly relevant to our study, as it alludes to the manner in which a sequentially

[15] Kenneth J. Conant, "The After-life of Vitruvius in the Middle Ages." *Journal of the Society of Architectural Historians* 27, no. 1 (March 1968). 33- 38
[16] Conant, "The After-life of Vitruvius," 34.

proportional method for ground plan layout expands an initial figure to create the whole and complete floor plan.

In 1968 in his article *The After-life of Vitruvius in the Middle Ages,* Conant argued against the idea that medieval masons were amazed at the idea of precise measurement. He assigned the term architect to men such as Gunzo and Hezelo, remembered for the design of the Cluny III under Abbot Hugh and he notes their high qualifications, Gunzo being *psalmista principius* (principle psalmist, a highly trained musician) and Hezelo referred to as *singulari scientia* (outstanding scientist). He says that the turning point in his study of Cluny III came upon recognition that the narthex "was proportioned like a Vitruvian atrium of the third class, which is a 1 : √2 rectangle (1 : 1.414) to which has been given the name "diagon."

However, in a 1975 article on the extensive research studies carried out at Cluny III from 1968 to 1975, he argued for series of symbolic numbers incorporated into the dimensions of Cluny as the determining principle of design.

> In our careful review, we found no dimensions of any significance which were not related to the basic quantities in three systems of acknowledged symbolic numbers...Within the Vitruvian norm of *proportio* and *symmetria* ... a dimension to be eligible for use in the design of Cluny III had to belong to one of three groups of allusive (that is, numbers alluding to something else) numbers."[17]

These three series were; 1. The series based on multiples or fractions of 100, 2. The series based on the year count of a solar cycle, that is, on multiples or fractions of 532, and 3. The series based on multiples or fractions of the number 7. Without any discussion of what the numbers in these series symbolically allude to it is difficult to assess them as

[7] Kenneth J. Conant. "Cluny Studies, 1968-1975." *Speculum* 50, no. 3 (July 1975).

governing factors in the development of a building plan. Conant notes that 532 feet, the solar cycle number, reaches from the center of the apse to the ashlar line of the west narthex wall. He treats 531 feet measured to the wall bench as also symbolic, because it is 530, the sum of perfect numbers, (6+28+496), augmented by the monad, that is, by 1.

Conant seems to have been of two minds regarding the factors governing the laying out of buildings. The article establishing Vitruvian geometry as a basis for laying out Cluny III was written in 1968 and his statement establishing symbolic numbers as the basis for the design of Cluny III is in a 1975 article. It appears he favored Vitruvian geometry before establishing the symbolic number series in his 1975 publication. It is his work on Vitruvian geometry that is relevant to this study.

Conant identified measurable squares in the dimensions of Cluny III and suggests the Romans used ‘onvenient approximations,’ often in series for related √2 values and (f) values that were surprisingly accurate. For (f) he gives the endless series 3, 5, 7, 12, 17, 19, 29, 41, 70, 99, 140 ... and for √2 the infinite series 3, 5, 8, 13, 21, 34, 55, 89 and 144....

To help us think not in terms of calculating solutions with numbers, but rather in term of using organized and manipulated spatial arrangements to solve problems, we can turn to an example known already in ancient Babylon. They noticed that when selected numbers are arrayed in an appropriate pattern, one can quite closely approximate the length of the diagonal of variety of squares as a whole number. Consider the array in figure three. Each number is the total of the two numbers immediately above. Thus 2 is the total of 1 and 1, 7 the total of 3 and 4 and 99 the total of 41 and 58. This array can be used to determine with considerable accuracy the diagonal of a square whose side is equal to any of the left hand two numbers in a given row of the array, that is, squares whose sides are 2, 3, 5, 7, 12, 17, 29, 41, and so on, numbers occurring in the left two columns. The number next to the right in any horizontal row of the array is the diagonal of a square the length of whose side is the previous number in the horizontal row. Thus 7 is a very close approximation of the diagonal of a square whose side length is 5. The length of the diagonal of this square whose side is five, by the Pythagorean Theorem, is equal to the square root of $5^2 + 5^2$, that is, 25 + 25 or 50. The square root of 50 is 7.071. The array indicates the answer to the problem is 7. The difference between the two solutions is only $^7/_{100.}$ The number 58 approximates the diagonal of a square whose side is 41, since the square root of ($41^2 + 41^2$) equals the square root of 3362 which equals 57.982. The difference here is slightly less than $^2/_{100}$ or $^1/_{50.}$. Squares whose sides measure 2, 3, 5, 7, 12, 17, 29, 41, 70, 99, 140, 169, 239, 338 and so on have diagonals whose length is very close to whole numbers and so this array can be used to find quite accurately the approximate length of the diagonals in squares of those sizes. These examples show how Babylonians used a spatial arrangement to secure numerical solutions to geometrical problems, keeping in mind that these are spatially configured solutions and not numerically calculated solutions. They also illustrate the indeterminate quality of such problem solving. It is not that the answers given are unacceptably inaccurate. Rather, the array can be used for only certain numbers, leaving

1	1	2
2	3	4
5	7	10
12	17	24
29	41	58
70	99	140
169	239	338
408	577	816
985	1393	....
....	....	....

Fig. 3. A Babylonian number array for problem solving

us in the dark about all the numbers missing from the array. It is this tradition of problem solving that connects to Conant's statement above saying that Romans resorted to "convenient approximations."

In Lib. VI, cap. II, I, Vitruvius says that nothing is more important than the choice of a principle dimension with which, for the purpose of *symmetria*, the dimensions of the parts have an orderly relationship. While Conant related the significance of this Vitruvian comment to the infusion of medieval architectural design with number symbolism and with an initial symbolic number that is the basis of symmetric relationship throughout the building, we take a different direction. We will show that in accord with Vitruvius, a principle dimension is expanded in harmonious relationships by plan-net geometry throughout the building not for symbolic purposes but rather for purposes of practical significance. We define the principle dimension as the length given to the side of an initial square. From this square, plan-net analysis will show a floor plan to be extracted from the square as a series of harmonious relationships by means of proportionally sequential steps using both the diagonal of subsequently generated quadrilateral figures and by copying intervals representing a side of these generated figures.

It is less than clear whether Conant ultimately stood behind the importance of number symbolism or behind the use of the diagonal of basic geometrical figures for the guiding principles of medieval architecture. Favoring symbolic numbers places Conant with architectural historians that sought the source of beauty in architecture. Favoring Manipulation of geometric figures places him with those seeking to solve the practical problems of architectural construction. Perhaps he is best considered as a transitional figure in the analysis of architectural design.

Rudolf Helm and Basic Geometric Figures

The German folklorist Rudolf Helm published from the 1930s through the 1970s. He was interested in the geometric basis for the vernacular architecture of the Nürnberg area. His working papers are archived at the Deutsches Kunstarchiv of the Germanischen Nationalmuseum in Nürnberg. His notes, drawings and photographs are of the highest quality. He sought to determine the fundamental geometric figures governing the plan and elevation of these vernacular buildings. Focusing on building height, width and length, he organized and compared the proportional relationships between these three factors. He published some of these ideas in *Das Bauernhaus im alt-Nürnberger Gebiet.* This was a republication of his 1940 work *Das Bauernhaus im Gebiet der freien Reichstadt Nürnberg*, the plates for which were destroyed in World War Two.[18]

Helm's notes record numerical calculations of the gross proportions (length by width) of floor plans. For these numerical calculations he assigned to a perfect square expressing a 1 : 1 ratio the value 1.000, which we will refer to as the squareness coefficient. He examined some eight hundred plans archived at the Nürnberg Staatsarchiv and selected some two hundred as sufficiently accurate for analysis. Of these two hundred, sixty-six are perfect squares, some sixty deviate from square by less than one foot and about eighty deviate by more than that. Based on these observations he noted that over time houses gradually transitioned from square to rectangular.

Helm proposed that plan element measurements were derivatives of basic geometric figures, the triangle, square, pentagon and hexagon. His thesis seems to have been that buildings were organized around the proportional relationships inherent in one

[18] Rudolf Helm, *Das Bauernhaus im alt-Nürnberger Gebiet.* (Nürnberg: Emil Jakob Verlag, 1978). The 1940 edition is available at some libraries. Illustrations, much the same, are larger, reproduced with higher quality and located near the relevant text rather than in an appendix at the end.

or more of the above fundamental geometric figures. Expressed in terms of the squareness coefficient these plans show clusters at certain numerical points along the scale. Thus, 1:1.0909 or 11:12 appears 10 times, 1:1.1111 or 9:10 appears three times, 1:1.666... or 6:7 appears three times and 1:1.142 or 7:8 appears two times. He suggested that 6:7 (1:1.666...) is sufficiently close to be associated with 1:1.54666... the proportional relationship of the smaller diameter of the hexagon (from side to opposite side) to the larger diameter (from angle to opposite angle). His analytical diagrams demonstrate this proposed relationship to the hexagon, pentagon rectangle or square. Plans derived from hexagons, he said, cannot be compared to square derived plans because the hexagonally derived buildings come much later in time and therefore they were much more accurately measured out.[19]

In figure four the hexagon diagram at lower left places the roof structure as a triangle within the floor plan that is shown as a rectangle rotated by ninety degrees to the actual floor plan to the right. The diagram shows that the wall lines constituting the rectangle coincide with the points of the six-pointed star. However, the bottom roof line at plate level does not seem to coincide with anything of significance. The diagram does effectively show the interrelationship of basic geometric figures, the circle, triangle and quadrilateral, in this case a rectangle, as intimately related to the proportions of the house. Helm's analytical method was to superimpose on the combined wall and roof lines a favored figure, in this case the six pointed star that defines a hexagon, but the analysis does not address the steps required to construct the floor plan, placing him with those seeking the source of harmony and beauty in architecture.

[19] Helm, *Das bauernhaus,* 37-38

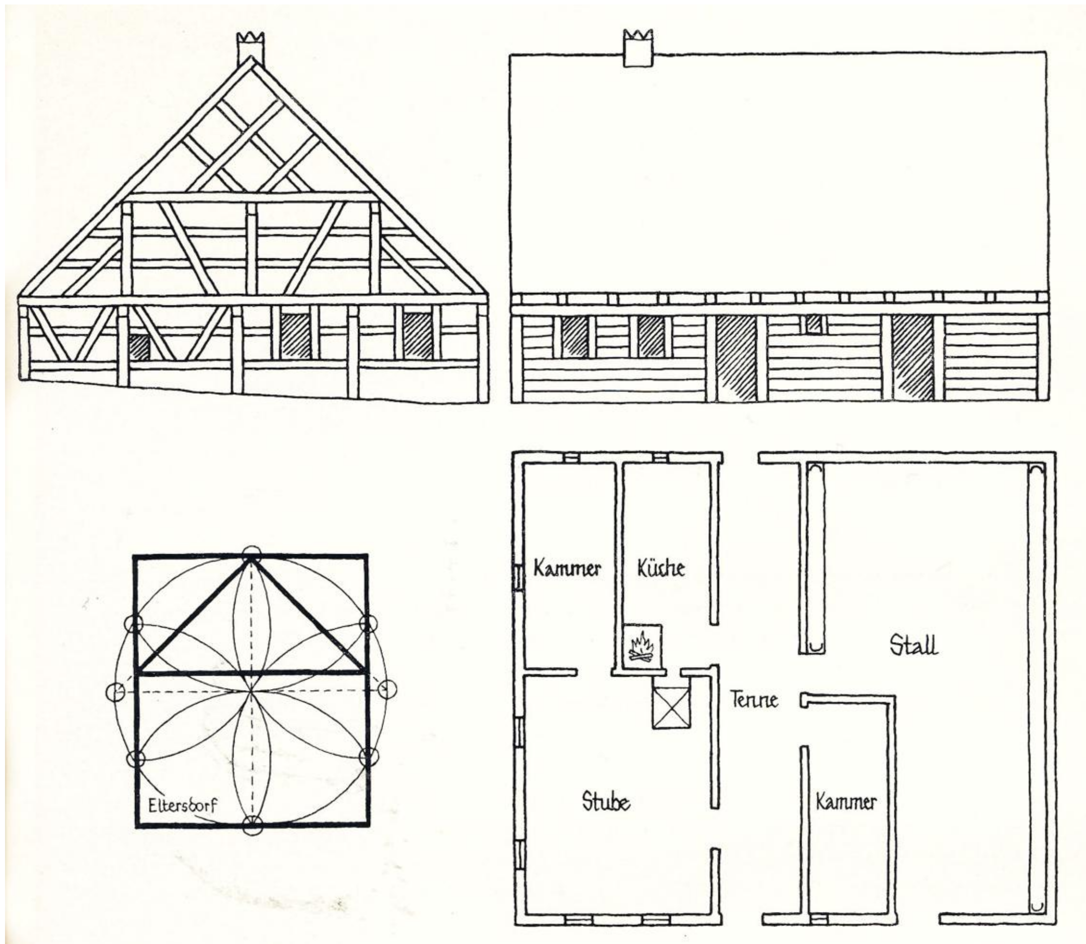


Fig. 4. Helm's analytical drawing of the Eltersdorf dwelling of Konrad Schultheiss, showing the hexagon (six sided star) diagram.

Helm does make a connection between geometry and elements of folk belief. "The connection between the square and the quadratic house plan is obvious," he said, "but that between the pentagon and the house derived from it is not."

> ...yes, it appears that in the outer structure the pentagon is not primarily evident, but that it nevertheless fully controls the construction work of the building. The formal meaning of the pentagon appears in this case to lie completely in the background within the magical. "The pentagram will give you much pain" says Faust to the Devil, who has found himself to be within the Drudenfuss and who will never again find his way out".[20]

[20] Helm, *Das bauernhaus,* 39

Without developing it as a working system of use to a builder, Helm suggests here that the use of the pentagon in governing construction could be attributed to a belief held over from the medieval period,

He does note that the pentagram or *Drudenfuss* (the witch's foot) has the quality of 'repeating its diagonal in the five-star,' and is thus able to 'grow or dwindle in unlimited greatness or smallness.' Here he implies that this property of the pentagon can be useful in the problem of scaling up from the construction drawing to the ground lines on the construction site though he makes no effort to detail how this was carried out in practice. He notes also that the pentagon

> "constitutes in itself the quality of the Golden Section... and so results in a system of parts in which at any time a part derived from the smaller cut demonstrates an interval related to that of the larger part in the same relationship as the larger one makes to an interval representing the whole."[21]

Again, he is describing a geometric property related to the scaling problem but without application to how it worked in practice. The implied reference here seems to be to the reliability that a pentagon based system will contribute to a geometrically based sequential system without however, developing these thoughts into a practical working system.

He suggested an evolutionary sequence is implied that develops from square to rectangle and occurred in a chronological series of steps from earlier to later and he concluded that such forms have evolved from less to more refined. He made the observation that because the hexagon is much less acutely angled than the square, it is "more gentle and equalized" in its proportions. He observed that floor to ceiling height is related to the physical size of humans and not to a geometrically derived factor and thus

[21] Helm. *Das bauernhaus im alt-Nürnberg Gebiet.* 38.

has nothing to do with the geometrical derivation of plan dimensions from fundamental geometric figures. He did not discuss how this observation impacts the organizational system that he sought to develop.

The Early Modern source material that Helm worked with and published is of great value and his drawings are the basis for many plan-net analyses. The source material consists of documents submitted to gain construction permits from the Nürnberg Forest Jurisdiction. Plans were required by the Forest Administration to assess the amount of timber to be allowed from the State Forest for the construction project. The collection is archived at the Nürnberg Staatsarchiv, the State Archive, and contains an estimated one hundred construction drawing of landed manor houses and around two and a half thousand ground and elevation plans for farm houses from the villages under Nürnberg jurisdiction. The quality of these drawings ranges from rough pencil sketch on a scrap of paper to carefully drawn and colored working drawings of professional quality. They range in date from the early seventeenth century into the first decade of the nineteenth century. At present they are physically organized in some four hundred and seventy boxes, interspersed throughout these boxes among other papers of every description. There is a catalogue relating geographic location to box numbers. The Staatsarchiv currently has a project underway to catalogue the documents and digitize them.

For all remodeling and reconstruction projects resulting from the many political and religious wars of the period, each submission had to contain a floor plan and elevation of the old structure and the new structure. For this reason an estimated total of some three to four thousand building plans total is not unreasonable. The buildings to be replaced could represent dwellings constructed as early as the beginning of the fifteenth century, and some do bear a striking resemblance to buildings portrayed by Albrecht Dürer, a resident of Nürnberg at that time. These documents generally give the floor plan and front and side elevation of both the old and the new building and generally contain building length, width and height, but do not give detailed measurements. In some cases there is a scale provided to take these measurements from the drawing with a divider.

Paul Frankl

Writing in the 1940s Paul Frankl and other writers we will review generally came down on the practical side, building on an observation that began to come into its own only some twenty years earlier. This view was that the current interest in the esoteric beauty of simple geometric figures in architecture obscured the strictly practical purpose of these figures in gothic architecture.[22] Medieval tradition persisting through the Renaissance, he says, suggested that simple, regular geometric figures were used as keys to proportion in architecture, figures such as the square, the equilateral triangle and the octagon. Frankl briefly outlined the development of interest in the practical rather than the artistic function of geometrical figures, noting that Matthias Roriczer had already published in this practical vein in 1486. The early Romantics he said endowed these simple geometric figures as visible instantiations of ‘extremely remote wisdom.’ Followers of the Romantics subsequently continued to overlay such geometrical figures onto plans and elevations. Frankl notes that some (overlaid) figures so complicated that ‘they could be used to prove anything at all.’

Opposition to Frankl’s approach arose early. In 1853 Carl Schnaase had defined art as the result of individual talent and irrational feeling, and nineteenth century critics of

[22] Paul Frankl, *The Art Bulletin,* 27 no. 1, (NY: College Art Association of America).

the Romantics ridiculed these complicated nets of lines, but all agreed in opposition to Frankl's viewpoint, that the 'mason's secret' was a magic formula of geometrical lines that guaranteed beauty.[23] The first to attribute a purely practical function rather than the production of beauty to the "mason's secret" was Georg Dehio in 1894. He demonstrated that the dimensions of ground plans and elevations of a number of cathedrals could be linked to the equilateral triangle. He assumed medieval masons did not make drawings and he doubted that they used a measuring stick.[24] Frankl notes however that Dehio was too attached to the idea of the inherent beauty of the triangle to develop a clear notion of its use as a practical tool.

He suggests that C. Alhard von Drach was the first to state that medieval architects used the equilateral triangle strictly as a practical devise with no thought of its aesthetic effect. Drach said that where building measurements are not commensurable one must assume they were established by geometry.[25] B. Kossmann in 1925 showed that the Cistercians used a pervasive measuring unit in their buildings that they called the "great unit" or *Grosse Einheit.* He did not attribute an aesthetic function to this, but he also gave no other reason for it.[26]

In 1929 in *Mittelaltliche Bauhütten und Geometrie*, Felix Durach noted both the lack of any common measuring unit in use in the medieval period and that no presupposition existed by which one could translate work from the scale of the planning drawing to the scale of the finished work using common standards of measurement.

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[23] Carl Schnaase, *Geschichte der bildenden Künst im Mittelalter,* (Düsseldorf: 1871), II, 223. Identical to the first edition, IV, 2, of 1853.

[24] Georg Dehio, *Untersuchungen über das gleichseitige Dreieck als Norm gotischer Proportione,* (Stuttgart, 1894), 20.

[25] C. Alhard von Drach, *Das Hüttengeheimnis vom gerecht Steinmetzengrund*, (Marburg. 1897), 5.

[26] B. Kossmann, "Einstens massgebende Gesetze bei der Grundrissgestaltung vom Kirchengebäuden." *Studien zur deutschen Kunstgeschichte,* no. 231, (Strasbourg, 1925).

Consequently some other method for accomplishing this transition in scale had to be devised. Durach writes

> The agreement between the design drawing and the construction at the building site consists therein, that the building master with his drawing employed the same principle of proportion as his people at the construction site practiced, only in natural size (Durach mean actual construction size).[27]

Frankl summarized by noting that proportion was used as a practical device to compensate for the lack of a common yardstick, and that Durach was correct to say that the choice of the proportion was not governed by beauty, but by simplicity for ever-renewed construction in different sizes. By 'ever-renewed construction in different sizes' Durach refers to the fact that sequential proportionality is not about absolute size since the same geometrical steps being taken, whatever the actual scale might be the drawing remained the same. Frankl also was correct to adopt the position that standardization of measurement did not exist in Medieval or Early Modern times.[28] Noting Durach's observation that the designer working on the plan and the builder working at the construction site used the same principle of proportion, we add simply that the builder did with cord and pegs on the ground exactly what the designer did with divider and straightedge on paper or parchment.

While Frankl made no mention of the term, it is of interest to note that the Ancient Greek principle of *dynamis* is at work here. *Dynamis* is power at work whereby some elements of the system change while other elements remain the same. When scaling

[27] Felix Durach. *Mittelaltlicher Bauhütten und Geometrie.* Stuttgart, 1929. 22.
[28] Arthur Lawton, "The pre-metric Foot and its Use in Pennsylvania German Architecture," *Pennsylvania Folklife.* 19 no. 1, (Autumn, 1969). 43-45.

up from design drawing to construction ground lines, the size and thus all real dimensions change, but the proportional relationships remain the same.

Frankl then described a case where pre-modern architectural designers struggled to resolve a major design problem without the use of highly developed calculating skills. He showed they did so by manipulation of geometric shapes. He reviewed briefly the controversy of 1391 over whether the Cathedral at Milan, started in 1386 under one plan by a previous building master, should be completed *ad quadratum* or *ad triangulum*. *Ad quadratum* was favored in Northern Europe and produced a much steeper roof pitch than did *ad triangulum,* which was favored in Italy for its lower roof lines. Frankl reviewed the issues and then commented briefly on the origins of these two design methods. He had already established the medieval understanding that 'all measurements must be commensurable or reducible to the same key figure' and that the purpose of medieval geometric methods was to enlarge the small design sketch or careful drawing to the size of the finished work. The plan-net analyses in subsequent chapters of the present work will be seen to follow these two principles specifically. B. Kossmann's *grosse Einheit* or Great Unit will help to understand the Milan Cathedral controversy and the solution proposed. The Great unit is the key unit to which all dimensions in a successful plan should reduce. In some buildings Kossmann found it to be seven feet and in others five feet, depending perhaps on the size of the local unit of measure. Frankl thought of it as a larger dimension that can be doubled or tripled and used as a very long rod for measuring larger distances during construction, especially vertical measurements at great heights. We will now see however, that it could also play a role in plan design. The controversy over whether to continue construction *ad quadratum* or *ad triangulum* arose with the departure of the first building master in 1491, some five years into construction. The building committee called the mathematician Gabriele Stornaloco in to determine under what plan to continue construction. He measured the construction work completed up to

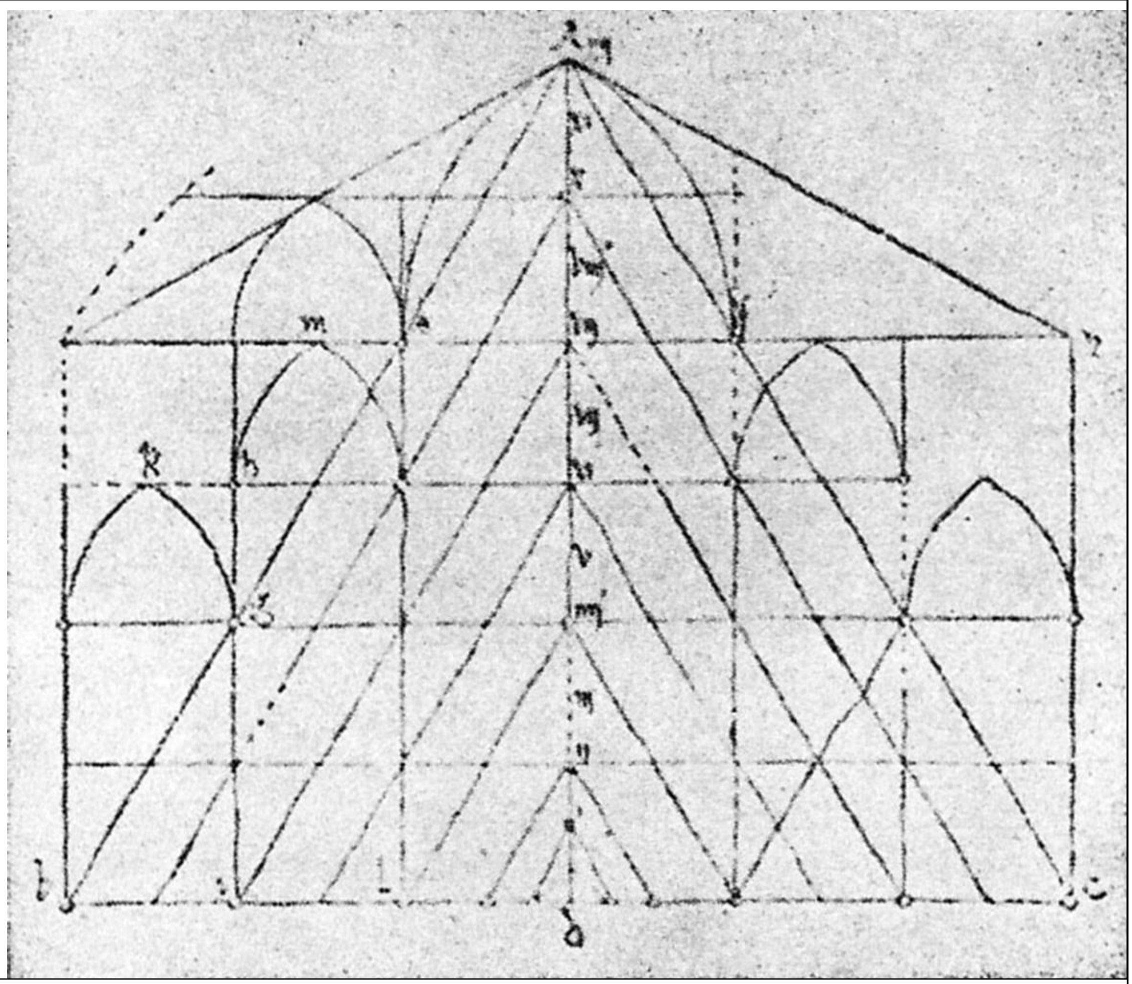

Fig. 5. Drawing by G. Stornaloco for Milan Cathedral, 1391 showing the rectangular network and concentric equilateral triangles.

that point and about a month later sent a letter containing an elevation showing his proposed solution. He fit the cathedral elevation within an equilateral triangle and thus his solution was *ad triangulum*. He divided the width and the height of the structure into six units. The divisions on the horizontal width were each 2 Great Units of 7 Milanese *braccia* long. This created a grid of rectangles, the corners of which were commensurate with nested equilateral triangles whose apexes were at the six division points on the vertical center line, a thoroughly *ad triangulum* solution. The vaults of the nave and the side aisles all were to spring from piers whose height matched the side lines of the various nested equilateral triangles. Stornaloco's proposal assembled rectangular geometrical shapes as elements of a larger rectangle as a solution. It reduced measurement to addition of small, simple whole numbers of the *grosse Einheit.*

The cross-section consisted of the central nave of two rectangles and two side aisles of one rectangle on each side, thus fitting the rectangle grid to the plan as 1+1+2+1+1, equaling six rectangles wide. Each rectangle was 14 Milanese *braccia*, or two Great Units of 7 *braccia* each.

Stornaloco's solution worked well in measurable whole units on the horizontal base line, but not on the vertical center line. Because the height of the equilateral triangle is incommensurate with the base of the triangle, the length and width of the rectangles that divide the base and the height by six are incommensurate in the same way, that is, they stand in relation to each other as $1 : {}^{1}/_{2}\sqrt{3}$. Thus the Great Unit does not fit to vertical dimensions expressed in whole units and so nothing is conveniently measurable, especially with the Great Unit at heights of up to 175 feet. The Milanese building committee made adjustments to the dimensions on Stornaloco's plan to reduce the ratio of length to width for his rectangles to 3 to 4. Since the length and width of the 3 by 4 rectangle are at right angles to each other, they constitute the base and the height of a Pythagorean right triangle whose three sides are multiples of 3, 4 and 5 and thus are measurable as whole units. Stornaloco, by thinking in terms of Great Units, was in fact designing in terms of repeated modules, thinking essentially in terms of computation that arithmetically manipulated very large measuring units seven *braccia* in length. Frankl used this design problem to show that if one was thinking in terms of measured units when enlarging the scale from the planning drawing to real construction size it was necessary to work with geometric figures such as the Pythagorean triangle that could be expressed in whole measured units, without requiring the manipulation of numbers to calculate relationships like $1 : \sqrt{2}$ or $1 : {}^{1}/_{2}\sqrt{3}$. The need to work only with terms expressible as whole units would put a severe restriction on what was possible.

Thinking about the square and the rectangle as the key figure extending harmonious relationship through the whole construction project, Frankl then made a distinction between Stornaloco and Matthes Roriczer. Stornaloco used the key figure of the rectangle to construct a grid, a system that Jay Hambidge would have classified as static. Matthes Roriczer used the square as the key figure from which to extract the completed whole in his booklets on cathedral pinnacles and gablets, a system Hambidge would have classified as dynamic. Stornaloco's method altered a "net of squares as used in the original plan" to a net of rectangles. In Roriczer's method the key figure consisted of two squares in which "the inner square is defined by one half of the diagonal of the outer square." He noted that in Roriczer's thinking, the measurements of the piers, the arches and the tracery etc. are all dependent on the first measurement, the side of the square, all other measurements being reducible in some way to this initial key measurement. Roriczer's practice he said is the secret of the masons, and it is rooted in Plato's *Meno* problem.

Stornaloco and Roriczer provide examples of solving problems of spatial definition and organization by means of manipulation of geometrical shapes.

If the designer and the builder were not the same persons, it was crucial that they use the same system of proportioning. Divider and straight edge and cord and pegs function using exactly the same geometrical principles. If they were the same person then the designer/builder used the same geometric principles in both designing and building. Frankl's conclusion gives strong support to the crucial function of sequential proportion in transforming a design from plan size to ground line size without the need for complex calculation. In essence, the method used to design a floor plan and the methods to lay out ground-lines are exactly the same thing.

J. J. Coulton

J.J. Coulton wrote extensively on Greek temple design in a journal article and as the subject of his book *Ancient Greek Architects at Work.*[29] He built a strong case that relates sequential geometric processes to the lack of drawing technology and practice in ancient Greek building arts. He noted the limited size of drawing materials available to the Babylonians and Egyptians. The former drew on raw clay tablets with a maximum size of about one third meter by one half meter. The tablet in the lap of the statue of Gudea, king of Lagesh is about one fifth of a meter by one third of a meter. For the Egyptians the maximum size of a sheet of papyrus was less than a half meter by one and one half meters. No Greek plan drawings are known to date. There is no mention of them in the literature, and no archeological remains of drawing instruments have ever been found. However architectural drawing and drawing equipment have survived from

[29] J.J. Coulton, "Towards Understanding Greek Temple Design: General Considerations," *Annual of the British School at Athens,* 70, (1975). and J.J. Coulton, *Ancient Greek Architects at Work,* (Ithaca: Cornell University Press, 1977).

Roman times[30] Lacking the technology for large measured and portable drawings Greek builders depended on an alternative system to produce the guiding dimensions and shapes as needed throughout the construction process.

He wrote in detail about the sequential nature of planning and transmission of design information concurrent with the erection of building noting that the only Greek preliminary design sources known are those in the form of written text consisting of specifications, as exist for Philon's Arsenal building. That being the case, he suggested that the architect would have to work out design details as the building went up. There is, he says, good evidence that this was the case. Thus, on a building constructed over a long period of time, late construction is in a late design style. Working in this manner as the building goes up begs the question "How did the pieces put down earlier transmit design information that came into play later in the construction process"? Coulton points out that this need arises early in the construction process, since the proportions of the stylobate (the platform on which the colonnade rests) affect the intercolumnation of the frontal and side columns.[31] Given that the Greek construction lacked drawings, Coulton considered the means used to satisfy the need for planning a building process as the building goes up to be characteristically Greek.

> Rules of proportion were formulated so that the appropriate size for each element could be derived from a dimension already decided....To Greek architects (the rules of proportion) were important in formal as well as technical matters, and in many ways the rules of proportion took the place of drawings as a means of recording design and of transmitting accepted norms over a large area of space and time. They also provided a means of predicting the appearance of a building before it was built....More

[30] Coulton. *Ancient Greek Architects at Work.* 53 and 68.
[31] Coulton, *Ancient Greek Architects at Work*, 54-59.

> importantly, the effect could usually be altered in a predictable way by changing a specific rule.[32]

Supportive of his suggestion that design took place as the building went up he compares this method to Vitruvius' successive system for the design of a temple in the Ionic order. In the Vitruvian system

> the rules do not relate each element to a simple common module, but they form a sort of chain so that each element is derived successively from a preceding one, usually the immediately preceding one. The ratios between successive parts are also more complex than in the modular system and the ratios between widely separated parts may be very hard to calculate. Because of this structure, such a system gives more scope for experimentation and variation, and so fits better with the existing evidence of Greek architecture.[33]

Even in a temple that lacked such careful refinement as entasis, that is, a slight bow in the lines of vertical and horizontal elements, each block for platforms and columns was specially cut and dressed for its own position and so shapes and sizes were not standardized.[34]

Though he did not speak of it as a one dimensional template Coulton suggested such a means for the solution of a problem in fitting stone to stone in temple construction that goes as follows. The outer Parthenon colonnade required properly angled beds in which to receive the two architrave ends that rest on each column. This is a problem because neither architrave end nor the intercolumnation distance was standardized. He suggested that the beds could be accurately and easily positioned by laying from capital to capital a straight wooden beam that is marked with the length of the appropriate architrave block. The relative area of each capital can then be dressed until each capital stone makes contact with the beam throughout. Solution by such a method exemplifies

[32] Coulton, *Ancient Greek Architects at Work*, 64-65.
[33] Coulton, *Ancient Greek Architects at Work*, 66.
[34] Coulton, *Ancient Greek Architects at Work.* 111.

architectural problem solving by manipulation of physical forms rather than manipulation of numbers, an example of an analogue solution, fitting shape to shape in the world of real objects.

Noting that there is little or no evidence for irrational ratios such as $1 : \sqrt{2}$, $1 : \pi$ or $1 : 1.618$ (the Golden Section) he suggested the difficulty in finding such simple ratios is due to the practice of approximating them to obtain ratios in whole numbers of feet. It is however, difficult to reconcile this with his observation that the stylobate of the Parthenon was constructed to a tolerance of 1 in 5000 parts, though in all fairness, he notes that column tolerances decline in some cases to 1 in 200 parts.

Coulton noted the limitation on size of drawing surface and lack of instruments as a significant basis for the use of successive methods in design and execution, as well as the obvious deficiencies of the Greek number system that for much of the time period had no means to write fractions in carrying out complex calculation. He pointed out the function of successive design in preserving and transmitting design detail by non-literate means. He demonstrated in detail a methodology that functioned through the manipulation of form and shape rather than manipulation of numbers. These factors can be taken as significant evidence for the source from which similar geometric methods were transmitted from the Ancient world to the Medieval and Early Modern periods.

Lothar Haselberger

Lothar Haselberger presented convincing evidence of the processes described by Coulton in an article in the journal “Scientific American” in 1985, entitled “The Construction Plans for the Temple of Apollo at Didyma.” The brief abstract at the head of the article claims that

> The nature of the 'blueprints' from which the Greeks built their temples has long eluded archeologists. A recent discovery shows that they were drawn on stone surfaces of the very temples they depict.[35]

The temple of Apollo at Didyma was a massive project started shortly after 334 BC and that dragged on for nearly six hundred years before it was suspended and never finished. Consequently many of the construction lines inscribed in the stonework are still visible, validating the practice of transmitting the information needed for the next step by means of the step just completed. In many cases these lines will have been covered over by additional stonework, especially floor markings indicating the placement of elements

Fig. 6. Construction lines inscribed on the surface of stonework at Didyma.

such as columns and walls as the base was built up to the level of the stylobate prior to erection of the temple proper. Where they were not to be covered, it was the practice of ancient Greek stonemasonry to leave the blocks sufficiently large that the markings were removed when they were given their surface texture.

[35]Lothar Haselberger, "The Construction Plans for the Temple of Apollo at Didyma," *Scientific American.* 253, no. 6, (December, 1985). 126-132.

Laurie Smith

In 2007, Laurie Smith published an analysis of the timber-framed porch of Old Impton, built at Norton in Radnorshire in 1542.[36] The door-head of the porch is intricately carved with foliage designs and an array of geometric symbols in the form of interlaced squares and interlaced arcs. She argues that the proportions of the porch are governed by a grid of perpendicular and diagonal lines generated from a module consisting of five interlaced circles and that this figure is contained in the decorative carvings incised on the porch. In figure ten from left to right I have rearranged the relevant drawings from her journal article to show the porch, the circular module carved on the door-head with the associated grid from which she develops her thesis, her

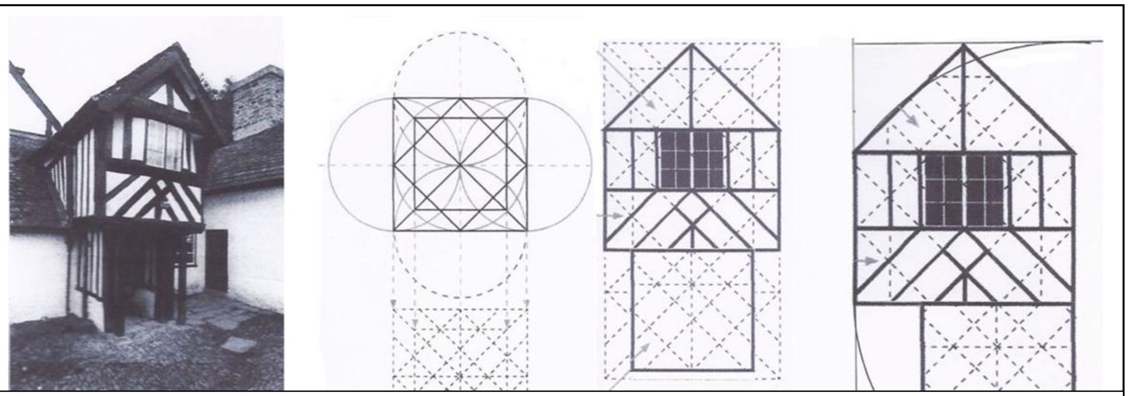

Fig. 7. Left to right, the Old Impton Porch, the five-circle module with grid, the grid applied to the porch dimensions and a plan-net analysis of the porch dimensions.

application of the grid to the dimensions of the porch, and my plan-net analysis of the dimensions of the porch. The grid is developed with vertical, diagonal and horizontal lines that connect the intersection points of the five-circle module. From the application of her thesis to the porch one sees that the lines of the structural members of the porch align with her grid lines. Smith notes that other elements of the porch are designed from

[36] Laurie Smith., "A Three Dimensional, Timber-Framed Encyclopaedia of Geometrical Carpentry Design," *Vernacular Architecture,* 38, (2007). 35-47.

the module of interlaced arcs found on the door-head, a six circle pattern she terms a daisy wheel.

While these complex geometrically constructed patterns could be laid out and used on a workbench or on a medieval tracing floor, their complexity and the size needed for building construction makes it difficult to see how the five circle and six circle modules could be effectively used at the large scale required to lay out an entire building plan on the ground at the construction site. The use of circles for elements of the elevation seems especially improbable, except when the construction project warrants a tracing floor where the circle modules could be laid out at appropriate full size and used as a template from which to transfer dimensions by cord or measuring stick to the work itself. All that being said, Laurie Smith has tapped into a very important aspect of the geometric methods and tools used in construction. At the smaller scale of individual parts and design of decorative details all that she describes seems quite possible at the carpenter's workbench as she makes it clear in titling her article as "Geometrical Carpentry Design."

Robert Bork

Robert Bork, writing in 2005 made a careful study of the use of manipulated geometrical figures such as the square and rectangle and the use of the diagonal of quadrilateral figures in the design practice of Gothic architecture. He attributed the connection between the craftsman as stonemason and the architect as designer to the stonemason's ability to design by creating drawings, a major technological advance in which at the design stage the building is thought of as a network of lines rather than as a sculptural mass.[37] He said that the draftsmen of the Strasbourg Cathedral in the decades around 1300 "used simple manipulations of the compass and straightedge to define the overall geometrical structure of their facade schemes" and that the entire drawing known as Plan B was based on a system of quadrature, or square rotation. Bork's comment that in the design stage the building was considered as a network of lines rather than a sculptured mass and that the network was accomplished by simple manipulation of the compass and straightedge was perceptive. This use of such geometry in the design stage worked to great advantage in a context of non-literacy because all the steps taken to design a complex plan on paper or parchment can be duplicated on the floor of the tracing hall with large divider and rule to create templates in real size for the stone carvers and then again by cord and peg in the construction stage by the masons.

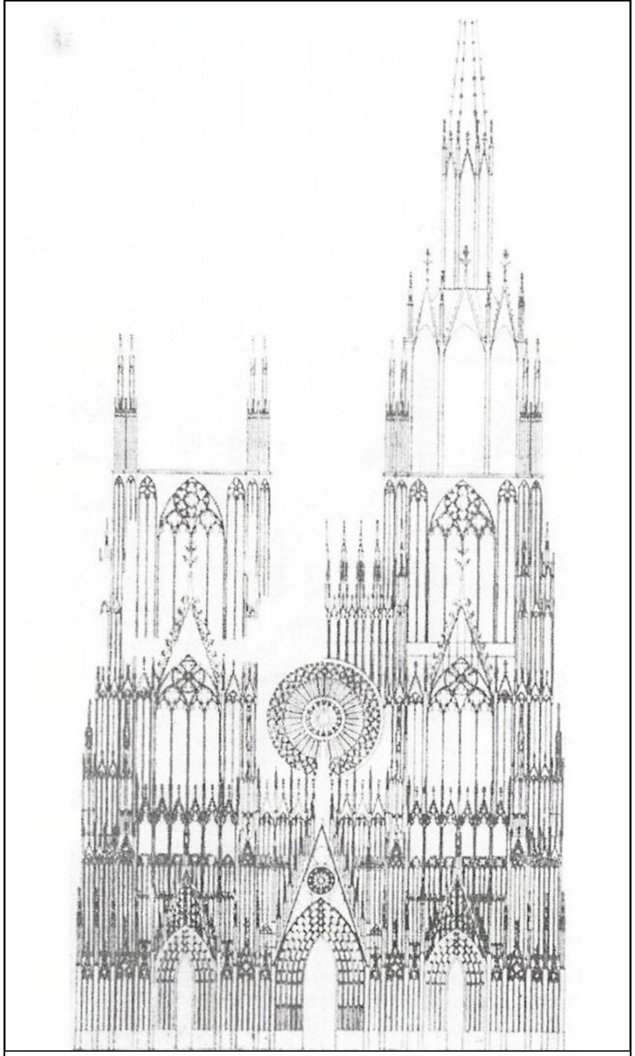

Fig. 8. Strasbourg Plan B with the surviving original drawing on the left and on the right as elaborated by G. Klotz between 1838 and1880.

[37] Robert Bork, "Plan B and the Geometry of Facade Design at Strasbourg Cathedral, 1250-1350," *Journal of the Society of Architectural Historians,* 64, no. 4, (December, 2000). 442.

He analyzed the tower zone of Strasbourg Plan B to show that its design is governed by a system of large rotated double squares of the kind that will become familiar to us when we look shortly at material from Roriczer's book on pinnacle design. Plan B exists now only as a partial design drawing, both the top and the bottom having

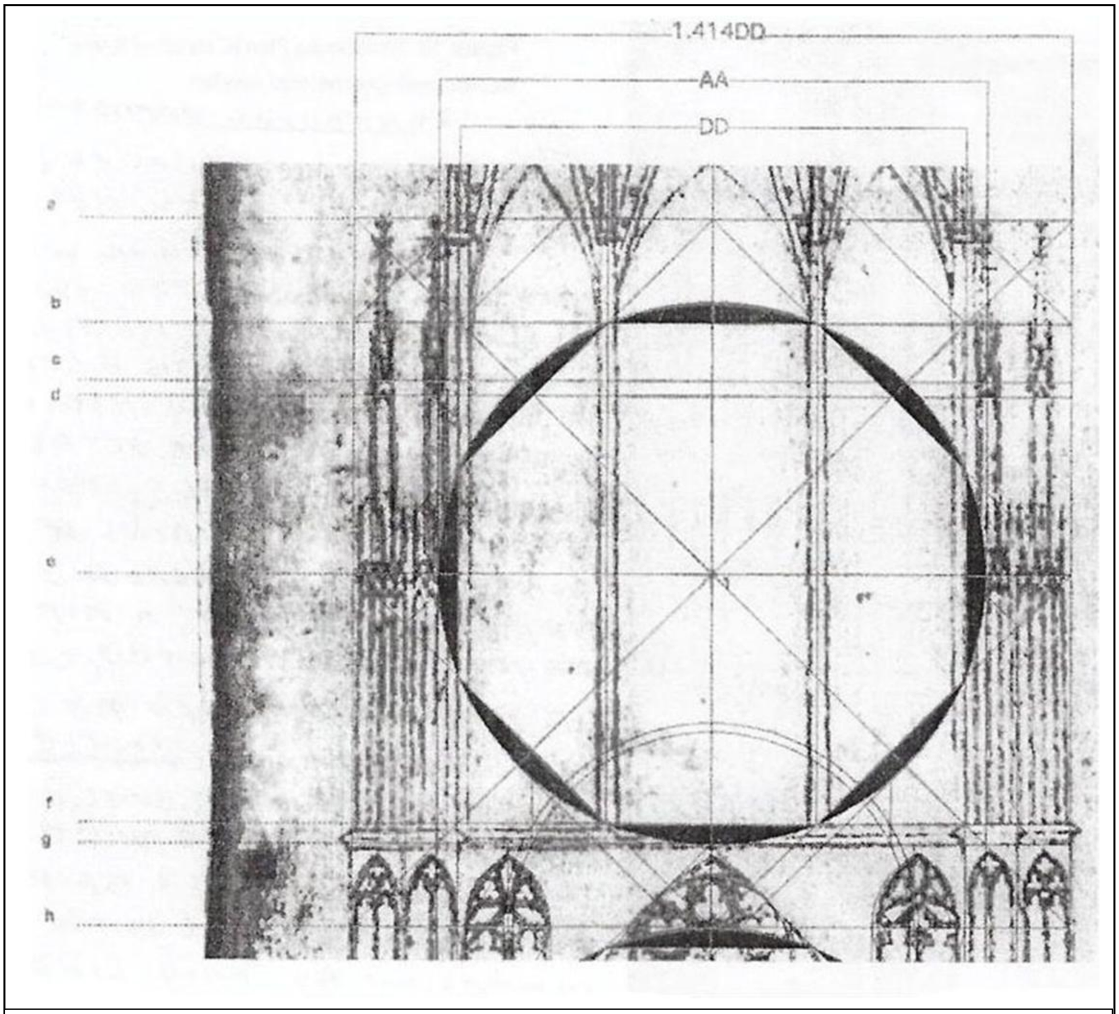

Fig. 9. Bork's generating octagon, inscribed within the two rotated squares familiar from Roriczer's book on pinnacle design.

been removed at some time in the past, and Plan B was not by any means the final design for the completed tower. However, Bork showed that a consistent original design concept built on rotated squares prevailed throughout the phases of tower design from plan B on the left half of figure four to the completed tower plan on the right half.[38]

The design process was hierarchically layered in the way that Schenkerian analysis and other analytical systems in music theory analytically divide music into background, mid-ground and foreground levels. The background level in Schenkerian analysis shows the harmonic movement from the tonic I chord to the dominant V chord and then a return to the tonic I chord at the end of the movement as skeletal structure. Within this broad skeletal structure are attached the mid-level elements that lead the musical experience from one part to the next. These elements might be musical phrases and periods perhaps held together by pedal point or an underlying Alberti base or ostinato, or modulation by various techniques from one key to a related key and back again, filling the segment with logical movement from the beginning to end. At the surface level are the decorative details, passing tones, neighboring tones, suspensions, doublets, triplet groups, runs and so forth that engage interest, provide variety and thus prevent boredom.

The hierarchical structure of such an analysis is a nested system. Applying this to an architectural facade such as that of the Strasbourg tower, the background level defines the constructions that set the broad form and structure of the tower. Nested within this structure are the mid-level elements such as doors, windows, arches, pillars, and so forth that give meaning to the assemblage and nested within the mid-ground structures are the foreground structures, the decorative detail such as window foliation, borders, finials spires, gablets, etc.

[38] Bork, 446. Illustration from Bork, *Plan B and the Geometry of Facade Design.*

Bork begins the analysis of the tower with a section of the Plan B drawing as seen on the left in figure eight above where it can be seen that the vertical rise of the tower consists of a consecutive series of squares. Within this tower section the background structure begins with a square rotated at 45 degrees whose apex is at line A. Line B forms the top side of the same square when rotated to the horizontal position. Inscribed within the star octagon formed by the two squares is a regular octagon around which is circumscribed a circle.

Horizontally, the line at A defines the height of the flanking pinnacles and the line at B the inset point for the finials on the pinnacles.

Vertically the two widths for windows are determined by the horizontal and two oblique faces of the octagon. In this manner the square, circular and octagonal figures determine the larger structure of the facade in this unit.

This is seen in figure ten where it is repeated up and down the tower. Mid-ground features are placed to conform to the background lines. Pointed arches and circular features over the sets of windows are placed within the upper corner of the

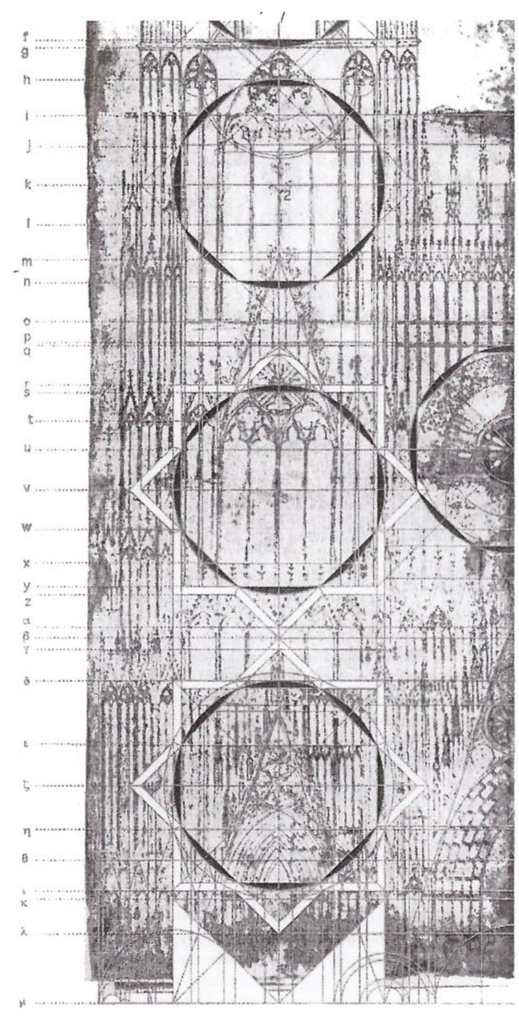

Fig. 10. The full set of generating octagons in the tower as described by Bork.

45 degree square, and windows are distributed symmetrically within and outside the octagons as mid-ground figures. These are bordered and embellished with foreground decorative details. Foreground details are then applied to conform to these lines. Diagonals of the rotated squares form triangles within which features are laid out in conformity to *ad triangulum* design. The squares themselves figure in *ad quadratum* design. This seems to be the reason that Bork stated that the two design methods worked together rather than as opposites.

He shows that the same system applies to the floor plan as applies to the elevation. While we will not follow the analysis in detail, in figure eleven we see the circle and the octagon in the same system of 45 degree squares. This is consistent with the principle stated earlier that all dimensional details must relate back to the initial module.

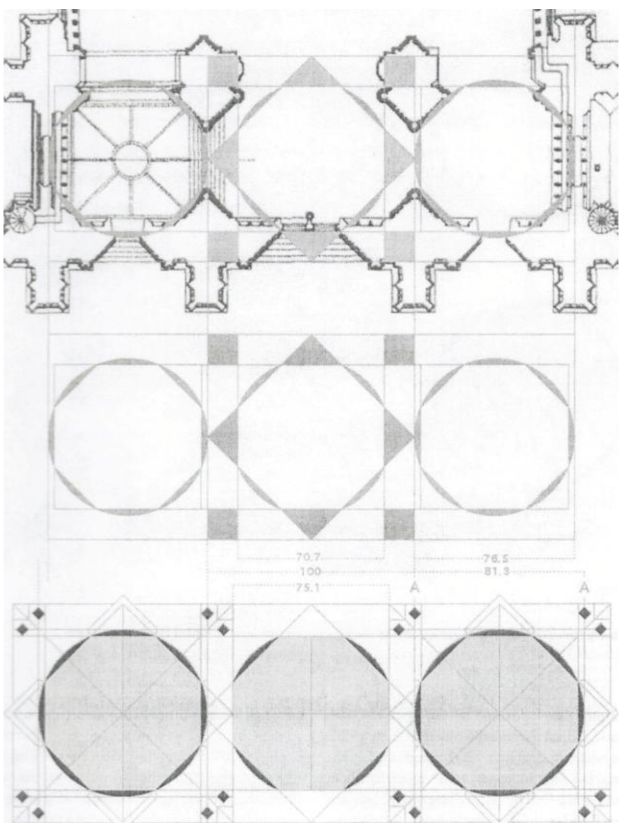

Fig. 11. Nested squares in the cathedral floor plan.

The matter of designing such a building as the Strasbourg Cathedral, or any other monumental structure makes evident how little we know about the steps between a drawing such as *Strasbourg Plan B* and setting the keystone in the ribbed and arched square roof over the transept where it crosses the nave of a cathedral. About these techniques we will find little by way of instruction in print. One might well ask why monumental architecture should be of interest in vernacular architecture. Folklore scholarship of a past era often referred to *gesunkenes Kulturgüter,* sunken cultural good, cultural materials that sank down from the level of high culture to be used at the everyday level. Others viewed cultural materials as crossing boundaries, these boundary areas being liminal spaces where ideas were mixed, matched and altered to suit a new circumstance. Regardless of how the theoretician addresses the issue, what was seen to be reliably useful toward the solution of a problem in the vernacular architectural world tended to be retained in the mind of the vernacular designer/builder.

Tons Brunés

If 'geometry by squares' is beginning to emerge in our thinking as a normative practice, we are beginning to see architectural plan design from a non-literate perspective. Tons Brunés published two volumes in 1967 that contain an evolutionary but speculative narrative of the development of geometric thought that is set in the context of an imaginary temple culture having the leisure to attend to the study of geometry and that held this knowledge secret from the common working man.[39] Though by no means established as historical fact in all their details, the evolutionary steps in his sequential development of geometrical knowledge are logical and plausible and so his thesis cannot be dismissed lightly. We will not examine volume 2 containing Brunés' application of the material in building analysis since his analyses do not address the issue of how, step by step, the practicing building master put the geometric material to use in order to lay out a

[39] Tons Brunés, *The Secrets of Ancient Geometry and its Use*. Vol. I and II, (Copenhagen: International Science Publishers, 1967).

specific plan. On the other hand the diagrams for his narrative in volume one reveal how usefully geometric constructions could be manipulated in the solution of architectural problems.

Brunés began with fundamental observations of natural phenomena. The circle was observed in nature, in the sun, the moon, the tree trunk and a circle grazed out by a goat tethered to a stake. The moon's cycle was observed to be twenty eight days and the sun's cycle equal to thirteen cycles of the moon. He suggested a vigesimal system based on twenty with fingers and toes to serve as the basis for counting. Supporting this are petroglyphs of a five-fingered hand above which are four lateral marks, implying four times five equals twenty. Brunés also noted circles incised on stone with a cross placed within the circle, and suggested five paces as the measure of each of the arms of the cross resulting in a circle whose diameter measures ten paces, the two crossed diameters totaling twenty paces.

In terms of a developing system of geometric thinking, Brunés associated the circle, square, triangle, circumference, diameter, radius, side and diagonal and hypotenuse as symbolically correlative to empirically observed natural phenomena and plausible cultural concepts. These associations he based on artifact evidence that supported his geometrical ideas, though the evidence does not necessarily support his proposed development of these ideas as a connected evolutionary process.

Having arrived at a circle, a square can be circumscribed around the circle by placing a rod equal in length to the circle's diameter tangent at the center of the rod to the circle and marking its end points with pegs. From one peg, the rod is again laid tangent to the circle at the rod's center and a peg place at its end. This is repeated two more times to complete the square. Brunés goes on to show that three concentric squares can be geometrically constructed that stand in a specific geometrical relationship to each other. These three squares are the basis for a large number of geometrically based processes that bear directly on the methods of the non-literate building arts. The sequence of geometric methods worked out by Brunés leads directly to the previously mentioned geometrical method of Matthes Roriczer in his booklets on the correct formation of pinnacles and gablets, and ultimately to the geometrical analyses of Welsh floor plans presented by J. Marshall Jenkins. The work of Tons Brunés, and that of Roriczer as presented by Lon Shelby will be examined in detail in the following chapter.

Lon Shelby

In the 1960s and 1970s Lon Shelby published a number of excellent journal articles that are carefully researched and highly informative. In a following chapter we will take a careful look at relevant material from his publication of the 1486 technical manual by Matthes Roriczer entitled *Büchlein von der Fialen Gerechtigkeit,* (Booklet concerning Pinnacle Correctitude), Roriczer's *Wimperbüchlein* (Booklet on Gablets) and Hans Schuttermeyer's *Fialenbüchlein* (Booklet on Pinnacles). He also wrote on medieval mason's tools, specifically the compass and square.[40] In *Speculum* he treated the geometrical knowledge of medieval masons in detail.[41] Here he described the process of quadrature, showing that elements of the square are manipulated to extract the elevation of an architectural feature out of a ground figure producing what Roriczer termed *ausgezogens Stainwerchs,* extracted stonework. Using drawings from the 1516

[40] Lon R. Shelby, "Medieval Mason's Tools," *Technology and Culture,* 62, (University of Chicago Press, Chicago, Ill., 1965) 236-248.
[41] Lon R. Shelby, "The Geometrical knowledge of Medieval Master Masons," *Speculum,* 47, no. 3 (July, 1972). 395-421.

Unterweisung (Instructions to my Son Moritz) of Lorenz Lechler, a Palatine *Baumeister,* Shelby showed how dimensions were transmitted by the divider from a base figure formed of rotated squares to determine the dimensions of the elements of a template for a window mullion.[42] This process he called constructive geometry, meaning the manipulation of geometric elements to solve construction problems with non-calculative methods. He noted Irwin Panofskys' observation that straight lines are guiding lines and not measuring lines.[43] Correctness of pinnacles (*Fialen Gerechtigkeit*) he said means that the elevation is extracted out of the base figure ((*Grund).* He quoted Hans Schuttermeyer whose last known date is 1518 and who was probably a goldsmith of Nürnberg who very likely crafted pinnacles and other architectural details in gold for elaborate table service.

> "This art (meaning quadrature) is truly planted and founded on the center point of the circle, together with its circumference of correctly set point and construction."

This brings us back to the circle of Tons Brunés and the square circumscribed around it, with all the diverse ways in which that figure could be used by the non-literate designer and builder.

J. Marshall Jenkins,

In a journal article published in 1967, J. Marshall Jenkins sought to understand the rationale governing the ground-rules of Welsh vernacular buildings.[44] He proposed that for the design of the buildings under consideration in his study, the principle methodological step was to swing the diagonal of a square to create a $1 : \sqrt{2}$ rectangle.[45] Following Sir Henry Wottan's 1670 advice in the latter's publication *The Ground-Rules*

[42] Shelby, Geometrical Knowledge," fig. 11 facing page 402.
[43] Irwin Panofsky, *Meaning in the Visual Arts,* (Garden City: 1955). 83.
[44] J. Marshall Jenkins, "Ground-rules of Welsh Houses: A primary Analysis," *Journal of The Society for Folk Life Studies*, 5, (1967). 69-91.
[45] Jenkins,"The Ground-Rules of Welsh Houses," 65.

of Architecture,[46] he concentrated on proportional relationships between elements of the circle, square and the “long square” or rectangle, especially diameters and diagonals. Wotton says

> Therefore, by the precepts and practice of the best Builders we resolve upon Rectangular squares as the mean between to many and too few angles, and which are through the inclination of the sides (which make the right Angle) stronger than the Rhomb or any other irregular square; but whether the exact Quadrant or the long square be the better, is not well determined, though I prefer the latter, provided the Length not exceed the Latitude above a third part, which would diminish the Aspect.[47]

Jenkins determined that Wotton recommended that on plan, rooms should be in the proportion of 1:1, 1:√2, 1:2 or 2:3, but that the external proportions of all the rooms taken together, that is, the building as a whole should be 1:1 or 3:4.

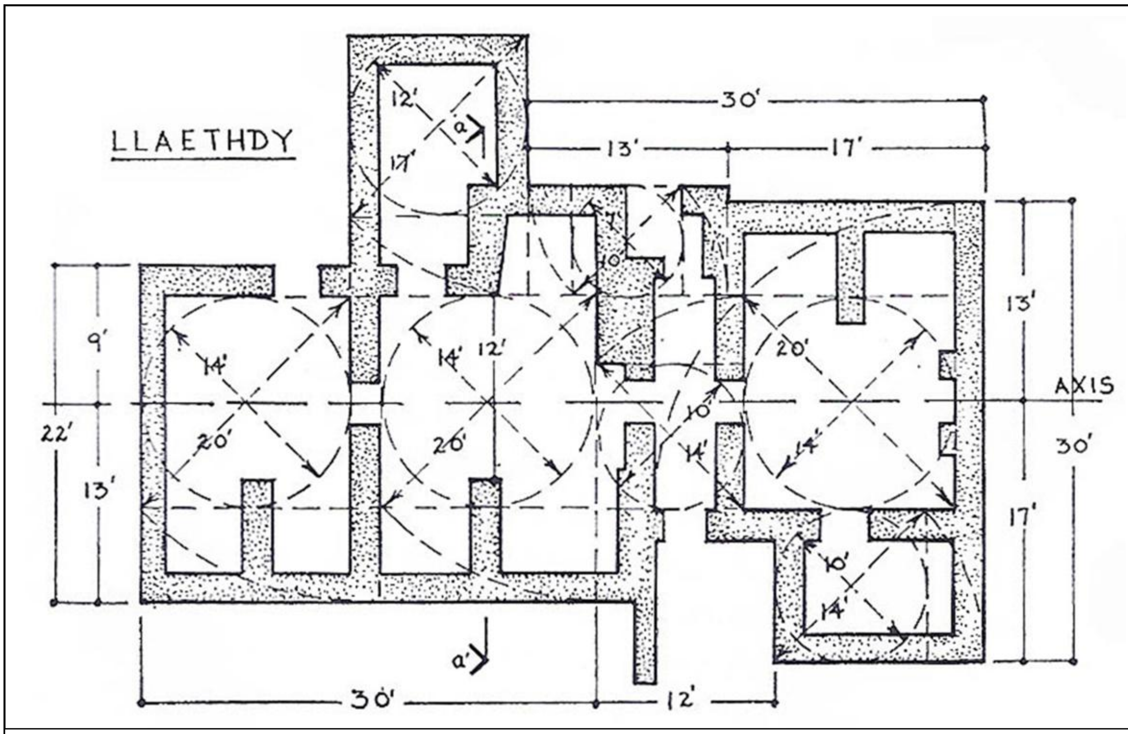

Fig. 12. Jenkin’s analysis by rotating the diagonals of squares that circumscribe an assemblage of circles.

[46] Published originally in 1624 under the title *Elements of Architecture.*
[47] Henry Wotton, *The Elements of Architecture,* (London: John Bill, 1624).

Noting the limited ability of “Pre-historic man” regarding numbers and measurement, he suggested that this had a determining influence on the early development of building practices. The only way the structural elements of a building could be related to one another is by geometry. He suggested the qualities of the circle as the starting point for elementary geometry and that the first building style was circular, and conical or hemispheric. He set forth the simple three step method with which a square may be defined by placing four diameter length poles tangentially around the circle, a method to be examined in detail in the next chapter. (See p. 63) Based on these relationships he suggested that the circle and its diameter, as well as the square and its diagonal formed the earliest and most important instruments for regulating building construction. As seen in figure twelve, by using these and a few other principles, Jenkins developed an analytical approach based on swinging the diagonal of the square and rectangle to generate the major elements of the floor plan. It will be noted in a later chapter that this method based on rotation of the diagonal is the basis for a floor plan design process that has the ability to extract the completed floor plan from an initial base figure in a sequentially proportional sequence of steps. Jenkins’ approach however, is an aggregation of steps adjacently located rather than an inter-related sequence of proportionally related steps resulting in a completed whole.

Summary

J. Hambidge distinguished between static and a dynamic symmetry that was understood by him as the process of rotating the diagonal of squares and rectangles to form root rectangles, the sides of which are incommensurable as drawn but that are commensurable when squared. Rotation of the diagonal is at the heart of plan- net geometry as proposed in this dissertation. Kenneth Conant recognized the pervasive presence of root rectangles which have been given names such as square, diagon, hemiolion, auron, dual diagon and embracing diagon. In the end it seems that his goal was to attribute symbolic significance to the numbers expressed in the dimensions of these geometric figures.

Rudolf Helm developed a system of geometrical analysis focusing not on monumental architecture but on the vernacular buildings around rural and hometown Nürnberg. He organized the gross proportions of buildings by length and width according to the degree to which they approached the 1 : 1 ratio of the perfect square. He held these proportions to be derived from such basic geometric figures as the square, triangle, pentagon and hexagon. He alluded to the pentagon's usefulness in scaling up from drawing to construction size because of the way in which the diagonal repeats itself in the five-star figure and is "thus able to grow and dwindle in unlimited greatness and smallness." As a folklorist he also noted the folkloric connection to protection from the devil entering the house. The devil, upon stepping into a pentagon figure, the "Drudenfuss," cannot step out of it again and thus is trapped forever in a house whose design is derived from the pentagon. He superimposed a geometric construction onto a plan or elevation but did not develop the ideas as a method for constructing a building design or laying out the ground lines. To his credit, his analytical constructions of the pentagon and hexagon figures conform well to the broad outlines of the elevations and plans of the houses and stables and barns he analyzed.

Paul Frankl was one of the first to view traditional medieval geometry in architecture as primarily practical in purpose. He summarized Felix Durach's 1929 argument that medieval designer and medieval builders used the same principles of proportion and understood that all measurements must be commensurable to the same key figure. This aligns with Matthes Roriczer's method of "extracting the elevation from the base figure" as well as the presently proposed process of plan-net geometry that expands the initial figure to the completed floor plan and ground lines. Frankl stated that the purpose of medieval geometric method was to enlarge a sketched design to the size of ground lines, a function we will demonstrate arises from the principle of successive proportionality. He commented on use of a net of squares and a net of rectangles in his discussion of the Stornaloco solution.

J.J. Coulton noted the mathematical and material limitations of the Ancient World that fostered the use of design and construction carried out by manipulation of geometric shape and form. He proposed that sequentially dependent steps were the means of controlling transmission of the design details from design to construction and that these sequential steps governed the construction process because details of the subsequent step were taken from the previous step. He compared this process to that described by Vitruvius regarding design of a Greek temple in the Ionic order.

Lothar Haselberger published evidence that the Greek temple builders did in fact use the material of the construction project itself to carry the information needed to guide in the following steps of the project. Laurie Smith developed a geometrical construction as the guiding principle for laying out the lines of the structural members of the Old Impton Porch.

Robert Bork observed that around 1300 AD the draftsmen of the Strasbourg Cathedral used simple compass and straightedge manipulation to design the geometry of facade drawings, and that Plan B is a system of quadrature or square rotation. He noted the hierarchical structure of facade design as a nested system and applied rotation of the diagonal of the square to explain some aspects of the facade and internal structure of the cathedral. On a much smaller scale, Laurie Smith argued that a 16th century timber framed porch was governed by a grid of perpendicular and diagonal lines generated from an initial figure consisting of five circles.

Tons Brunés developed a descriptive narrative of the evolution of geometric knowledge applicable to those issues at the heart of the dissertation. His diagrams match many basic figures in Matthes Roriczer's book on the design of pinnacles. They shed considerable light on a practical methodology for the use of the square inscribed and/or circumscribed with a circle to solve architectural problems without numerical calculation. These proportional relationships occurring in the square inscribed with the cross, the eight-pointed star and what he terms as the 'sacred cut' were used to solve many of the problems facing the Pre-Modern designer/builder in a world of very limited calculating ability. The procedures reconstructed by Brunés were among the non-literate designer/builder's working tools and methods. Understanding their use gives significant insight into now forgotten patterns of thought in the process of architectural design and building.

Brunés applied his geometric tools to the analysis of buildings in his 1967 publication on ancient geometry and its use. These analyses do not address the issue of successive proportionality. They are geometrical diagrams that are superimposed on top of building facades and plans to find the points of coincidence between elements of the diagram and the facade or plan. As with so many analyses of historic architecture, Brunés fails to give any sense of the means by which design of the facade or plan was thought out and constructed but in his narrative of the development of geometrical tools he has outlined clearly the logical continuity informing many of the methods available to the Pre-Modern designer/builder.

Perhaps the earliest to develop an analytical approach that allows demonstration of the methodology for developing a building plan by purely geometrical means was the 1967 publication by J. Marshall Jenkins. The basic figures of his analyses are the root rectangle as defined by Hambidge and the diagon, dual diagon and other variations of rotating the diagonal of the square as named in Compton’s work. Jenkins understood this as method derived from the primitive hemispheric or conical hut that defines a circle. Similar to Brunés’ proposal for circumscribing the circle with a square, Jenkins bases his analysis on the rotation of the diagonal of a square that was constructed in the same manner. His diagrams of analyzed floor plans are accurate and account well for the dimensions of the buildings. Like Helm, he did not account for the factor of successive proportionality and so his analyses, accurate though they are, constitute an assemblage of individuated steps associated by position relative to one another rather than a sequence of step that are organically unified by proportion because they unfold in sequence from an initial base figure according to a proportional constant.

Two core ideas came into focus from the transition in academic thought described in chapter two. One is recognition of the use of the root rectangle generated from the square for marking out space that in its external proportions and its internal divisions is unified in a single system. The other is use of an equally unified system of sequential proportionality to solve problems of scaling changes and of transmission of construction information. There are four key elements to be examined in the following chapter that contribute to understanding the use made of the fact that in a system transition, some elements change and other elements remain the same; 1. Stasis and change in system transitions, 2. Manipulation of the elements of the square and circle to define relationships useful in the building crafts, 3. The use of root rectangles and sequential proportionality to extract a complete figure from an initial square base figure, 4. Application of the extraction process to pinnacle and gablet design.

The following chapter notes how in the *Meno* dialogue Plato empirically demonstrated the ancient Greek concept of *dynamis* that they believed empowered stasis in some elements of a transition while permitting change in others. A reconstruction by Tons Brunés of the early development of geometric thinking demonstrates the inter-relatedness of the circle, square and triangle. These inter-relationships serve to make the square a fundamental element in the methodology of non-literate architectural problem-solving, that is to say, solution by manipulation of geometric figures rather than arithmetic numbers. Richard Tobin's reconstruction of the methodology of the Canon of Polykleitos provides a clear visual demonstration of the idea of sequential proportionality and the use of root rectangles as a means to generate a completed whole figure from an initial square. The uses of the square described by Brunés are an essential part of the method for extraction of concrete architectural figures from an initial square as developed in a mid- fifteenth century booklet by Matthes Roriczer on the rectitude of pinnacles and gablets. In closing the chapter examines a textual description from the Sulva Sutras of cord and pegs problem solving for the lay-out of massive and ritually precise altars in Vedic India.

Chapter III

Root Rectangles and Sequential Proportionality

As mentioned briefly in regard to the work of Jay Hambidge, one observation carefully studied in the ancient world and especially by the Ancient Greeks, was that in certain systems some things remain the same as other things change. The Greeks called the power that enabled this phenomenon the *dynamis.* Plato used the word extensively and with a number of meanings in various contexts, one of which was to express ideas about the nature of proportional relationships in sequences involving change.[48] The ancient Greek music theoretician Aristoxenus understood *dynamis* as a function and on occasion, specifically as the power within a system to preserve intact some elements of the system while other elements changed.[49] This ancient idea bears on the present work. In a sequentially proportional design construction process, when some elements are transferred unchanged to the following step, they form the base figure from which new elements are generated. This is the *dynamis* principle at work, The unchanged elements from which new elements are generated function here as template, as instructions for the next step. I use the term sequential proportionality to render the occasions where this idea of *dynamis* is present in the design and construction process.

To demonstrate the presence of *dynamis* underlying a process of change we look at a problem regarding change in the size of a geometric figure that was posed by Plato in the *Meno* dialogue between 394 and 390 BC. In Plato's narrative Socrates shows how the size of the solution figure relates directly to the size of the original figure through

[48] Joseph Souilhé., *Etude sur le Terme DUNAMIS dans les Dialogues de Platon.* (Paris: Libraire Félix Alcan, 1919). 23-25.

[49] For examples of Aristoxenus' use of *dynamis* as a function, see *Greek Musical Writings, Vol. II: Harmonic and Acoustic Theory* edit. Andrew Barker, New York: Cambridge University Press,1989. 150-156

proportionality. Socrates' intent in posing this problem is not relevant to our purpose but his method of solving the problem is. The following is a short synopsis of the *Meno* problem.[50]

Socrates drew a square on the ground and asked his companion Meno to bring one of his slaves to view the square on the ground which is subdivided into four smaller squares so that the whole figure is two units on each side. He then asked the slave to draw

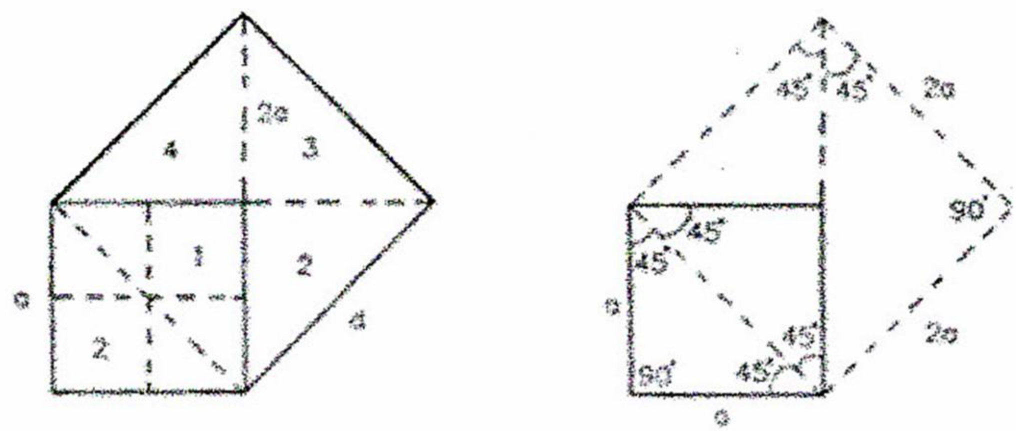

Fig.13. Plato's Meno problem, demonstrating that the larger square contains 4 triangles of the same size as the two triangles in the smaller square and is thus double in size.. Drawing by the author.

a square that is double in area, telling him that if he cannot calculate it, he shall demonstrate it without calculation by drawing the solution out on the ground, that is to say, by manipulating the geometric figures instead of manipulating numbers by calculation. Guessing at first that the solution will be to double each side, the slave boy creates a square four by four, but this results in an area of 16 smaller squares, which is four times the original rather than double the original. The slave then guesses that it must be 3 on each side, resulting in an area of nine squares and is wrong again. Socrates then

[50] Plato, Meno. *The Collected Dialogues of Plato, Including the Letters.* edit. Edith Hamilton and Huntingron Cairns, Princeton: Princeton University Press, 1971. 64-70.

shows him that if he draws a square on the diagonal of the original square whose area is four as indicated in the left hand diagram in figure thirteen, the new square will be double in size. Socrates empirically demonstrated an observable proof of the fact that the new triangle is twice as large and no more or no less.

The diagonal divides the original square into two equal portions. When a square is constructed on that diagonal one triangle constitutes half of the original square and the balance of the new square divides into three additional equal sized triangles. Each of these triangles is half the size of the original square, and because the new square contains four such triangles, it is double the size of the original (4 X ½ = 2). The diagonal is the geometrically constructed mean between the original square number and a square number equal to an area exactly twice the size of the original square. The diagonal is the unchanging element and the old and new squares are the changing elements of a system undergoing transformation empowered by the *dynamis.* In the plan-net geometry that we will develop a little further on, the proportional relationships between the parts being generated constitute the unchanging element and the actual dimensions of the parts are the changing element.

Acting as actual or potential elements of each other, the interrelated circle, triangle, square and rectangle have been universally used in problem solving by non-literate means. To understand architectural plan design and construction processes from a non-literate perspective we must understand these inter-relationships. We will first examine how these interrelated geometrical figures developed a form useful to the designer/builder, then the use of sequential proportionality in design, followed by extraction of an architectural elevation from a square base figure and to close, a concrete example of the use of cord and peg geometry in architectural design and layout.

Tons Brunés published two volumes in 1967 that contain an evolutionarily organized narrative of the development of geometric thought set in the context of an imaginary temple culture having the leisure to attend to the study of geometry and determined to keep this knowledge secret from the common working man.[51] Though the steps in this process are by no means established as historical fact, his thesis of a logical sequence of developing geometrical knowledge cannot be lightly dismissed. His analyses do seek to put his geometric figures to use in understanding the form of the buildings but as may be seen in figure fourteen, they do not address in a practical way the problem solving issues faced by the building master. On the other hand the diagrams for his developmental narrative in volume one do reveal how mankind could have come to understand the inter-relationship of the above mentioned basic geometric figures, and how these relationships were useful in geometric constructions manipulated to solve architectural problems. It is of interest then to understand how these inter-related figures are constructed. Many similarities to

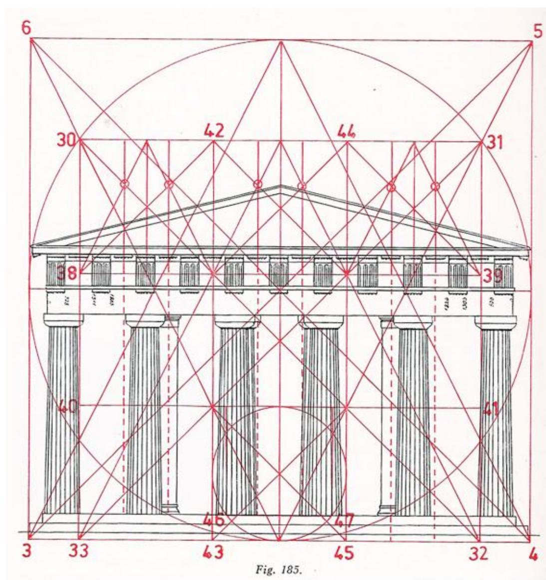

Fig.14. Geometric analysis of the Parthenon facade by Brunés

[51] Tons Brunés, *The Secrets of Ancient Geometry and its Use*. Vol. I and II, (Copenhagen: International Science Publishers, 1967).

interrelated figures will be evident later in the chapter when examining Lon Shelby's reproduction of the diagrams of the fifteenth century *Baumeister* The drawings that follow are adapted from Brunés by the author. Noting that the circle is a form observed in nature, Brunés began with empirical observations of natural phenomena to establish the circle and the square. He said that the concept of radius derives from the practice of defining a circle by extending intervals of equal length to left and right, and forward and backward to form a cross from the center point. Selecting the number five to represent the fingers on each hand, he suggested the length of each interval be five, making the diameter of the circle equal to ten. In figure fifteen a line AC of ten units in length is laid out with a cord and pegs. To place the intersecting line BD, AC is divided in half using the geometric construction to bisect a line. Once the vertical intersecting line is placed at the center, the interval from the center point to A is copied up to B and down to D and the

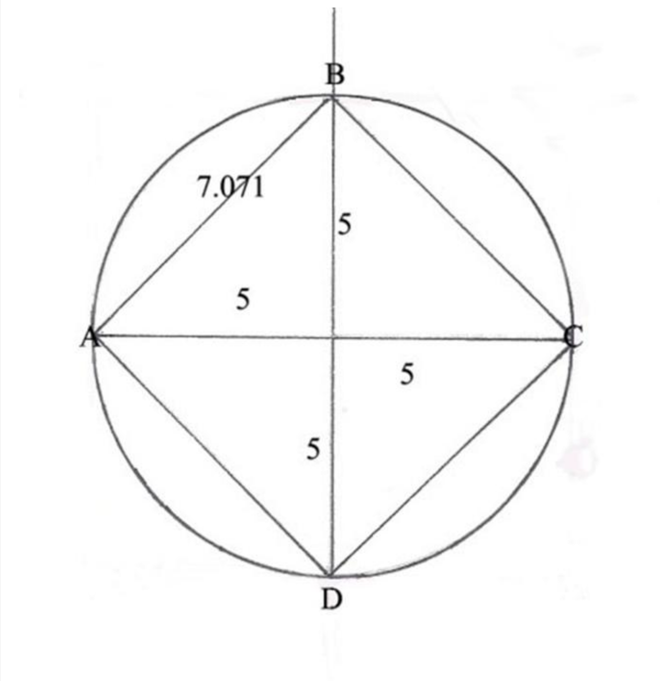

Fig. 15. Brunés' circle and cross showing related triangles, square and circle.

lines connecting A, B, C and D form the inscribed square. Connected together at A, B, C and D they also form four triangles two of whose sides are five and whose hypotenuse is very close to seven, actually 7.071. The four lines AB, BC, CD and DA form an inscribed square whose perimeter totals very close to twenty eight paces, which Brunés suggested was associated with the lunar month.

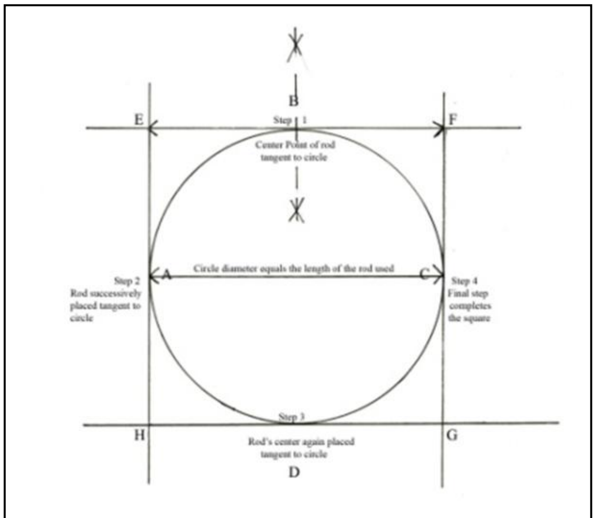

Fig. 16. The center of the rod placed tangent to the circle on four sides forms a perfect square.

In figure sixteen, a square circumscribing the circle is formed by placing the diameter AC or BD tangent to the circle at the center point of the diameter, and this is repeated three times to form the circumscribing square. In figure seventeen the circle with its inscribed square ABCD, the four radii, the intersecting diameter and the 5-5-7 triangles are the inter-related geometrical figures that will form the basis of much of the material that follows. In terms of the development of geometric thinking, Brunés associated the circle, triangle, square, diameter, circumference and radius of the circle, the side and diagonal of the square and the sides and hypotenuse of the triangles as correlative to

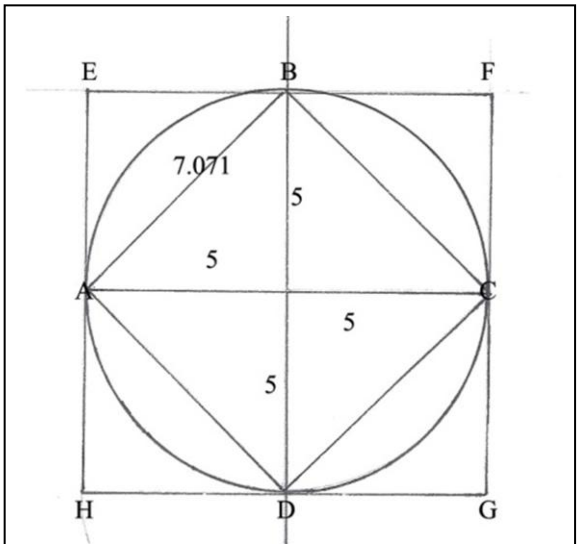

Fig. 17.Circle with inscribed and circumscribed squares and triangles.

empirically observed natural phenomena or as logically derived from them. He based this on artifact evidence supporting his geometrical associations, though the evidence does not necessarily support his proposed evolutionary narrative.

In figure eighteen a series of three concentric squares may be constructed as follows. The mid- point of each side of the outer square, at the point where the side is tangent to the circle is marked A, B C and D and these four points are connected by diagonals to form a new, smaller square rotated forty five degrees to the outer square. The diagonals of the outer square, EG and HF, intersect the four sides of the smaller square at their mid-points. These points are marked and then connected to form the small inner square that is in parallel orientation to the outer square. When all lines are taken together they divide the large outer square into twelve smaller squares and twenty four triangles.

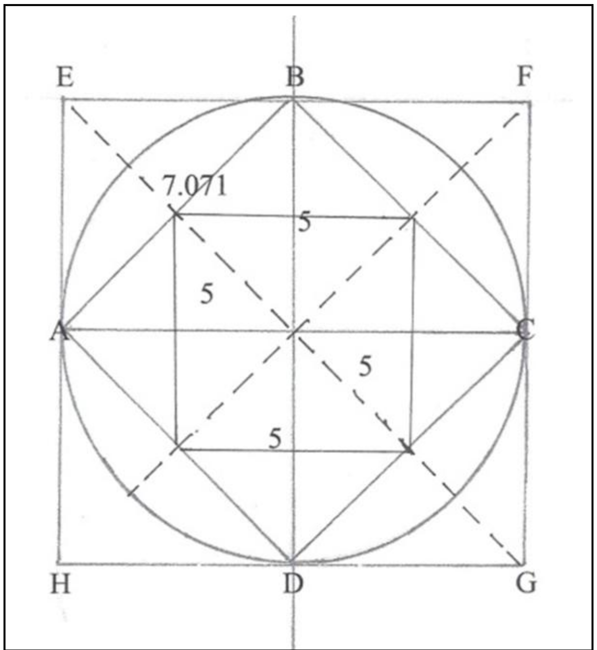

Fig. 18. All lines taken together divide the outer square into sixteen small squares and thirty two equal triangles.

As illustrated in figure nineteen, these three squares can be arranged in parallel by taking the following steps. Construct the cross consisting of lines CA and BD with arms of equal length. Connect points A, B, C and D to form a square. Mark the mid-point of lines AB, BC, CD and DA using the construction to find the mid-point of a line segment. Designate these intersection nodes as points (a), (b), (c)and (d). First using (d) as the pivot point and then using (a) and (d) as pivot points, swing two arcs whose radii are equal in length to (da) and mark the point where the arcs intersect as point F. Similarly, from points (d) and (c), (c) and (b) and (b) and (a), swing intersecting arcs to mark points G, H and E. Connect E, F, G, and H to form the middle square. Find the mid-points of lines EF, FG, GH and HE in similar fashion. Where these lines cross EF, FG, GH and HE, designate these mid-points as (e), (f), (g) and (h) and draw in lines (fh) and (eg). From (e) and (f) swing arcs to locate point I as was done for the middle square. Continue in the same manner to find points J, K and L to mark out the outer square. Squares arranged in this way form Matthes Roriczer's base figure for the erection of a pinnacle. They are proportionally related because the length of the diagonal of each square equals the length of the side of the next larger square. In this sense they are sequentially proportional.

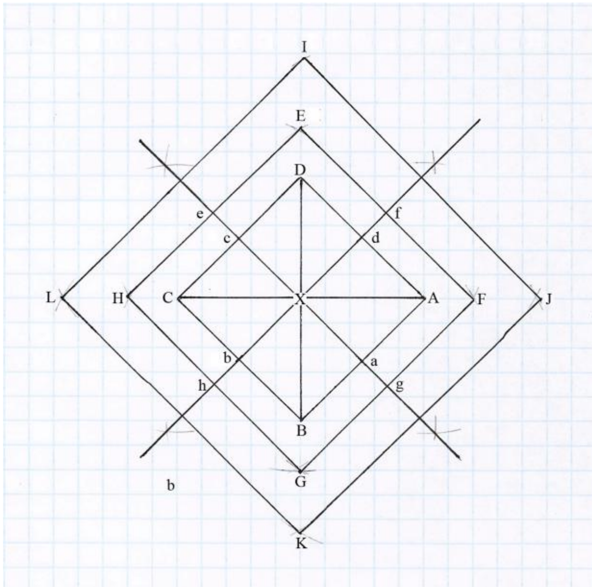

Fig. 19. When all squares are in parallel, they conform to the diagram given by Roriczer for the extraction of a pinnacle body from the base figure.

These three concentric squares can be organized to share a common corner rather than concentrically sharing a center point. In figure nineteen they share a diagonal in common from L through H, C, A and F to J. In figure twenty the cut termed by Brunés the *'sacred cut'* lies between the inner and the outer square. The *sacred cut* square is not in the same proportional relationship to the inner and outer squares as the middle square is in figure fourteen. To construct the two outer squares of this construction the same step is taken successively in two sets, swinging the diagonal, whether CA, CG or CJ, to the right and to the left. To construct the next square lines EG and FG are drawn parallel to the sides of the inner square. To arrive at this figure the following steps are taken. Construct cross ABCD as before. Then connect A, B, C and D to form the initial square ABCD. Extend CB, CD and diagonal CA beyond the size of the largest square desired. Rotate diagonal CA to E and then to F. From E and F draw lines EG and FG parallel to sides DA and BA. Points C, E, G and F mark the square representing the *Sacred Cut.*

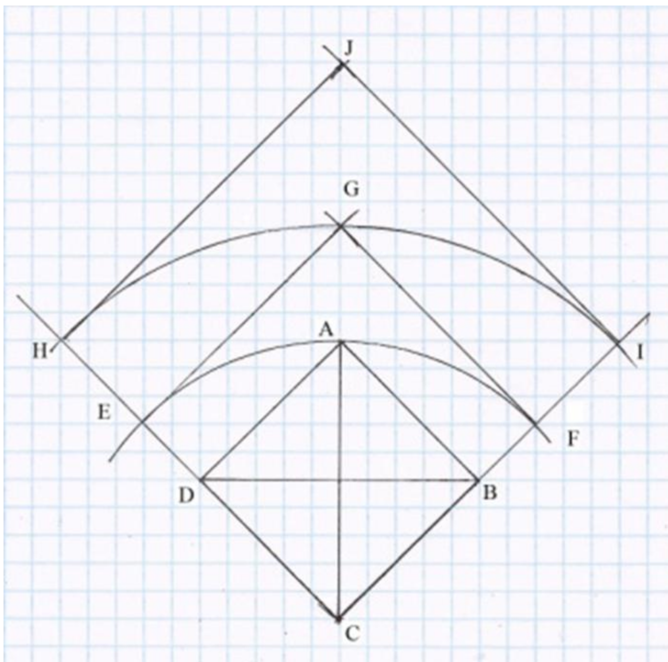

Fig. 20. Similar to Fig. 11, the square is expanded as the *Sacred Cut* at EGF and to the next step at HJI.

To double the length of the sides of square ABCD to that of JICH which has four times the area, follow the same procedure, swinging diagonal CG to H and to I. As in the previous case, from points H and I, draw lines HJ and IJ parallel to sides EG and FG. This forms the third square, sides twice as long but with an area four times that of the first square. This can be verified by counting squares, for which the graph paper background is provided. Speaking with numbers, if side CD of the inner square is equal to 1, then by the Pythagorean Theorem diagonal CA is equal to √2, and thus side CE of square CEGF is equal to √2. By the same theorem side CH of the outer square is the same as diagonal CG of square CEGF and by the same theorem equals $(\sqrt{2})^2$ which equals 2.

Brunés viewed this as a division of two squares, the largest square JICH with sides double the length of the smallest, ABCD. He called this division by the middle square the *Sacred Cut.* Swinging the diagonal AC of square ABCD in one direction to E or to F to produce a root rectangle, or swinging it in both directions to produce a larger square are the basic steps of design by plan-net. Our intent is not how this figure relates to the far more sophisticated design figures represented in Mattheus Roriczer's book on pinnacle design or whether it represents an example of *gesunkenes Kulturgut* or how it fits into some other theory of transmission. It is to establish that such methods were part of the general understanding available for vernacular building construction. The most straight forward statement is that it worked with simplicity and effectiveness and that vernacular builders recognized that the root rectangle enabled them to lay out proportionally related intervals on paper, parchment or tracing floor and then to transmit them as ground lines at construction scale onto the building site. This process allowed a design to be marked out on the construction site without previously having been set out as a drawing as long as one could remember a set of repeated and simple steps carried out in a specific sequence. While the steps remain the same the completed plan derived from the initial figure is a unified whole regardless of the scale at which it is constructed.
In all three cases the set of nested three squares contained in figures eighteen, nineteen and twenty are the same relative size in relation to one another but arranged differently in each case to serve different purposes. Thus the proportional relationship one to another of intervals copied off the nested squares is the same regard regardless of the size of the

figure. As long as the whole constructed figure remains a square, intervals relate one to another in the relationship of the side of the larger figure to the diagonal of the smaller figure.

There are a number of other uses to which the square as a diagram can be put. Plan dimensions must often be divided into smaller units, as for example, the divisions of cathedral windows, the placement of columns or the arches along a wall. Marked cord lengths copied from a square diagram on the tracing-floor or on the ground near by the construction site can be used as templates to transfer dimensions, establishing those points with less likelihood of error than if they are laid out by repeatedly placing a measuring rod down to measure out the intervals along the line. As illustrated in figure twenty one a square into which the eight pointed star and the sacred cut are inscribed can be used to divide the side of the square into 2, 3, 4, 5, 6, 7, 8, 9, and 10 intervals. If the diagram is being used as a template from which to copy measurements to the construction work, only three lines are necessary to determine

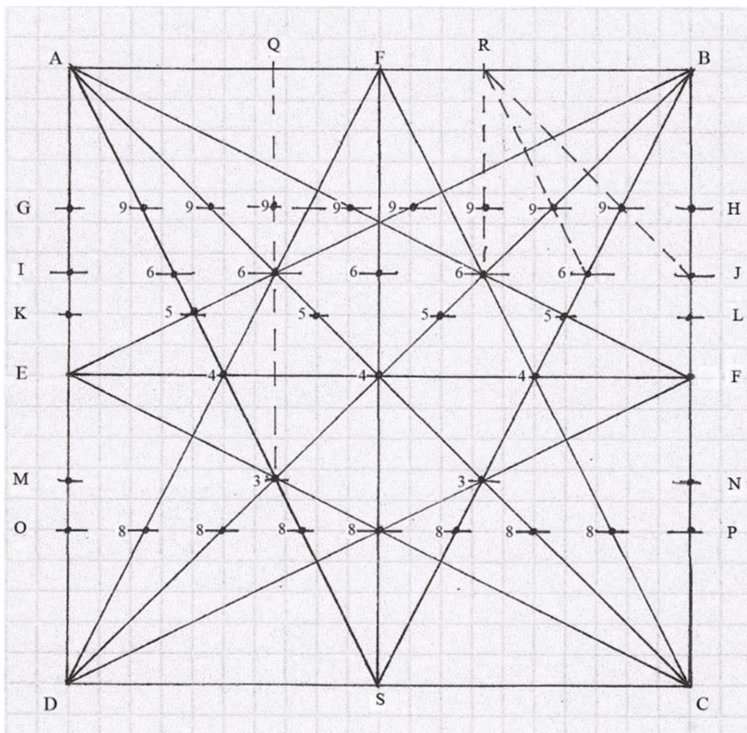

Fig. 21. The square, the eight-pointed star and the cross provide intervals determining the division of the square’s side into 2, 3, 4, 5, 6, 7 8, and 9 segments. (Drawing by the author, based on Brunés).

the interval to be copied. To get one ninth of line AB, one needs only lines AD, GH and AS. To mark an interval equal to one fifth of AB one needs to have only lines AD, GH and AS. It is necessary, however, to have in memory the position of all of the lines in order to know which lines to select for cording off each interval lengths desired. While remembering all of this would be a formidable requirement for the modern mind, it is not out of the question to a mind for which much of culture was transmitted from memory.

The cross and the eight pointed star inscribed within the square are sufficient for dividing a line into two, three, four, five, six and eight intervals. To find division points for seven intervals the sacred cut must be inscribed in the square according to the method noted in figure thirteen and to divide into all nine intervals an additional simple construction must be worked into the diagram using points of intersection in the eight point star. Division for 10 intervals is done using the lines of the eight point star, but two of the intervals thus found must be halved, the simple geometric construction for halving a line segment sufficing for this.

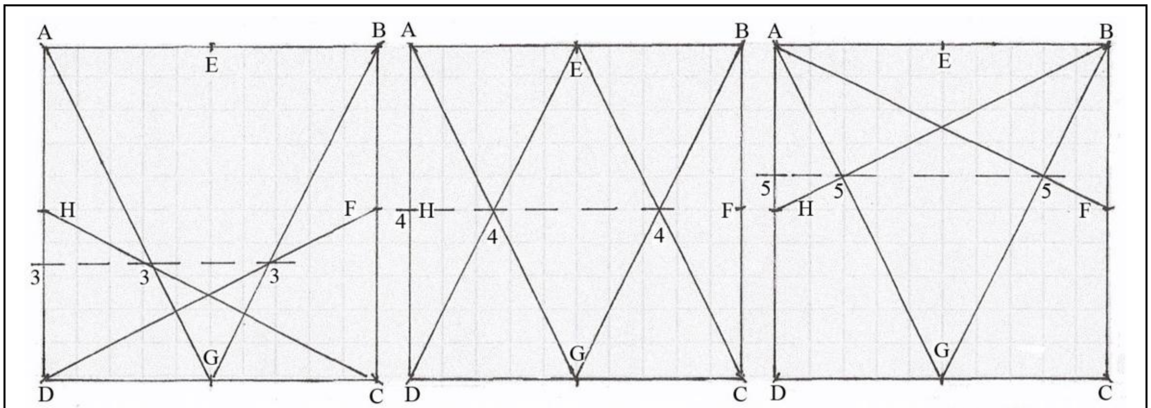

Fig. 22. How to find intervals of 3, 4 and 5 with a minimum of lines. Drawing by the author.

The simplicity of using this diagram is apparent in figure twenty two, showing the method for finding intervals of 3, 4 and 5. By duplicating the step on both sides of the square the horizontal dashed lines can be marked that establish permanently staked out positions on the perimeter to be used in conjunction with the intersection lines to determine interval lengths. Only two intersecting lines are needed for each interval length. The apprentice learns and remembers that all three procedures use the triangle ABG and the first uses the lower rectangle HFCD and the last the upper rectangle, ABFH. Interval lengths determined on this square can then be copied by cord to the ground lines and marked with pegs. This would be particularly useful in determining long distances when laying out column centers and building them to the same height. The columns support the roof arches so they must be placed accurately and of correct height if the arch is to be structurally sound. In the welter of construction scaffolding a very long measuring rod would be difficult to use to assure equal column heights to the spring points of the arch resting on the columns. That distance would be much more easily determined if copied from one column, marked on a cord, carried to the opposite column and then stretched upward. This method reduces the probability of accumulative error but it does necessitate long cords. We will see later in this chapter that the miniature of Gunzo's Dream shows Ss. Peter and Paul manipulating a very long cord, and St. Stephen with the unused portion of the cord coiled over his shoulder.

We have seen here a rigorous and logically connected derivation of ad quadratum figures based on the interrelated circle, triangle, square, and the sacred cut that are eminently useful in the geometrically solution of architectural problems. Brunés' modern thinking on how such geometric figures were guarded and taught are highly speculative,

but the geometrical relationships that his diagrams demonstrate are not speculative at all. Examining them and experimenting with their use gives insight into the practical use of geometric figures as in Richard Tobin' reconstruction of the Canon of

Polykleitos, the Sutras of the Cord used in Vedic India to lay out the construction of variously shaped altars and the material published by Matthes Roriczer in the fifteenth century for extracting the elevation of pinnacles and gablets from the base figure in what what Lon Shelby calls constructive geometry.

The Canon of Polykleitos to which we now turn applies a single principle throughout a series of sequentially proportional steps to expand a base figure into a completed whole. Each step of this successively proportional process used the pattern and shape formed in the preceding step to transmit the information needed to shape the geometrical form of the following step. Such steps may have been drawn informally on the ground, traced on the plaster floor of a cathedral tracing hall and soon erased, or incised into stone blocks such as those seen on the unfinished Greek temple at Didyma in Turkey, but fated to disappear upon finishing the stone's surface or when covered over by subsequent construction.

Richard Tobin's reconstruction of the Canon of Polykleitos, is a straight forward application of the principle of sequential proportionality. Polykleitos was the most illustrious Greek sculptor of the High Classical Period and the author of a book on the ideal proportions of the male figure entitled *Kanon.*[52] The problem he tackled was to breathe life into the hard marble and bronze of sculpted humans and animals. One of his contributions to this task was the relaxed and fluid S-shaped contrapposto stance, and in his book he determined the proportions of each of the elements of the sculpted human body. Κανονίζω (kanonizō) is the verb meaning to measure by rule or to regulate and κανόνις (kanonis) was a rule for measuring.

[52] John Griffiths Pedley, *Greek Art and Archeology,* (Upper Saddle River, NJ: Prentice Hall, 1998), 265.

In a unified way the canon served to determine harmonious dimensions for each part of the human body to be reproduced in a statue. This was accomplished by an unchanging dimensioning principle (*dynamis*) common to each body part expressed in two principles; 1. The representation of three dimensional elements is manipulated by means of the canon as a two-dimensional geometrical figure, either that of a square or a rectangle. The proper measurements of the human body parts are taken as the length and width of each geometrical figure produced by the canon. The lower arm from the wrist to the elbow is understood as a rectangle having length and width and likewise the upper arm from elbow to shoulder. 2. The second characteristic is that sequential proportionality governs expansion from a single initial base figure into a fully expressed whole. For example the upper arm rectangle is generated out of the lower arm rectangle because its length is taken as the length of the diagonal of the lower arm rectangle. Richard Tobin's reconstruction of the canon and its application to the Doryphoros statue by Polykleitos shows how successive proportionality can govern the creation of the whole from an initial base figure.[53]

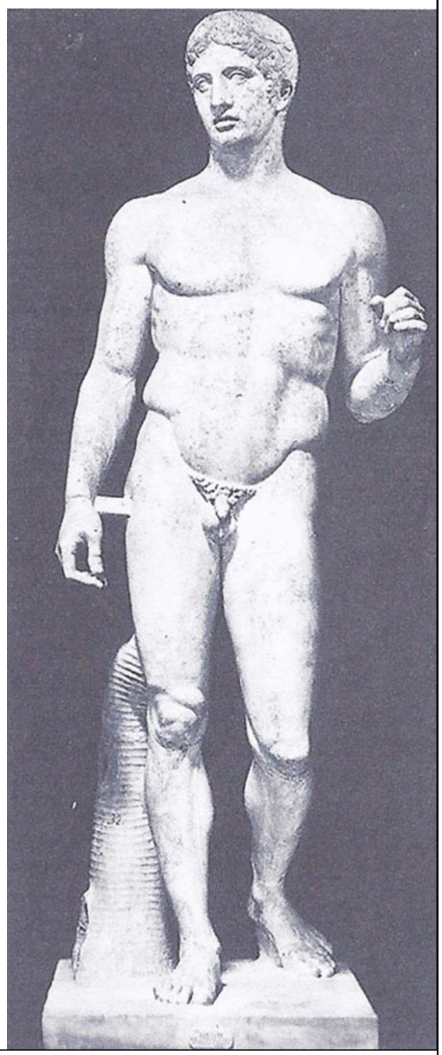

Fig. 23 Doryphoros, meaning spearbearer, by Polykleitos. National Museum, Naples.

[53] For a full treatment of this proposal, see Richard Tobin, "The Canon of Polykleitos." *The American Journal of Archeology.,* 79, no. 4 (Oct., 1975), 307-321.

The Doryphoros statue in figure twenty three is a Roman copy of the Greek original sculpted by Polykleitos, found in Pompeii and now at the National Museum in Naples. Richard Tobin proposed a reconstruction of the canon method and compared its results to the actual measurements of the Naples Doryphoros. Tobin illustrated the process in figure twenty four. He took the distance from the tip of the distal phalange of the little finger and squared it, geometrically producing a square each side of which was equal to the distal phalange length. The diagonal of that square, longer than the side in the ratio of 1 : 1.4142 is the information needed to take the next step determining the length of the medial phalange.

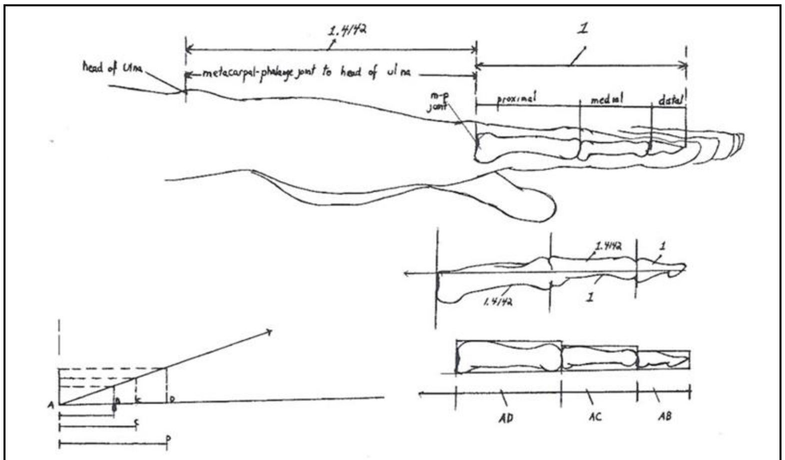

Fig. 24. The canon of Polykleitos applied to the forearm. (Drawn by Richard Tobin).

The diagonal of the medial rectangle is rotated to form a new and longer rectangle by the same method. The diagonal of this newly constructed rectangle determines the length of the proximal phalange adjoining the knuckle. Thus three consecutive rectangles are constructed, each longer than the previous one by a factor of 1.4142.... The canon allows however, for slight variation in the way it proceeds as may be seen in the next step in which the three rectangles thus far produces are consolidated into one as the base figure for the next set of steps.

These three rectangles are then taken as a single rectangle equal in length to the entire little finger, and its diagonal is then rotated to create a rectangle whose length is the distance from the metacarpal-phalangeal joint, the proximal knuckle, to the head of the ulna at the elbow. The diagonal of this rectangle is used to generate a new rectangle that determines the length of the upper arm, ending at the top of the shoulder, the acromium.

The process is now shifted to the body proper in figure twenty five. From the top of the head to the highest interval of the arm is taken as the distance from the top of the

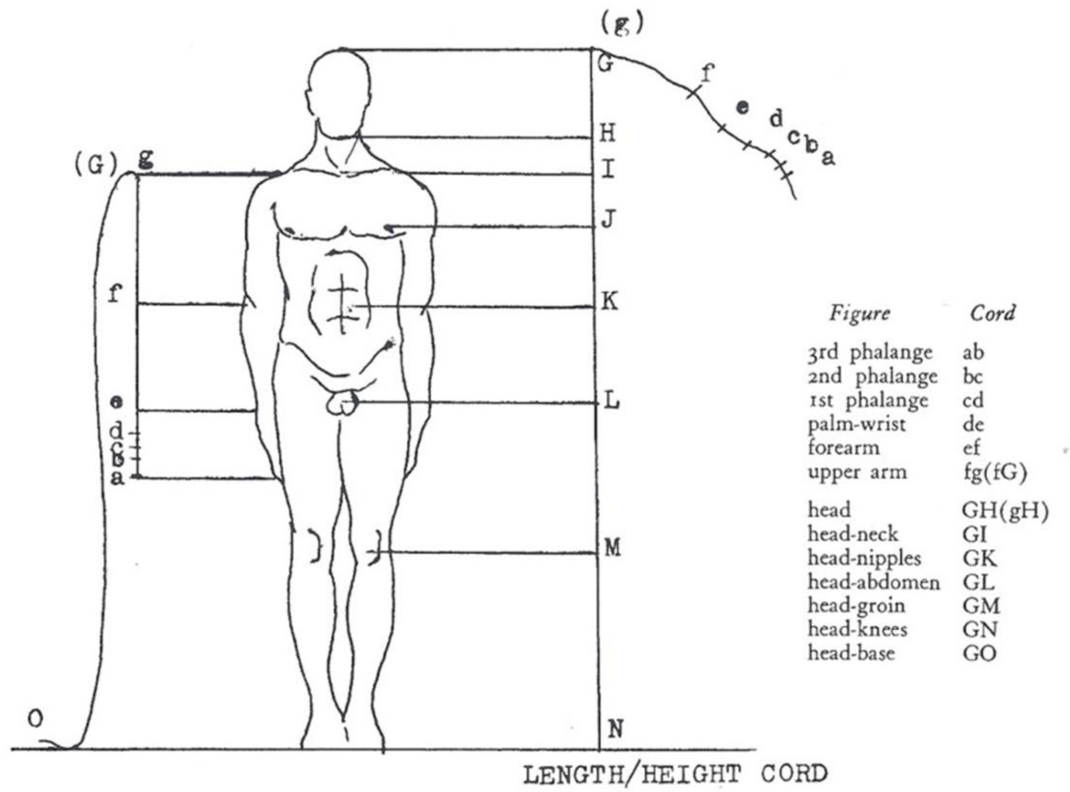

Fig. 25. The canon applied to the whole body. (Drawn by Richard Tobin)

head to the juncture of the clavicle, on the same line with the acromium of the shoulder. The diagonal of the squared interval from head to clavicle extends from the top of the head to the nipple, and the diagonal of the head to nipple interval extends from the head to the abdomen just above the navel and on a line with the elbow. The quadrangular diagonal from the head to abdomen length extends from the top of the head to the groin and the diagonal of the quadrangle from head to groin gives the head to knee distance. Finally, the diagonal from head to knee extends to the soles of the feet.

Tobin has demonstrated a number of things of interest. 1.) He showed that when dimensions are represented as elements of geometric figures in two dimensions rather than as numbers they can be manipulated as geometric forms to achieve artistic and practical results. 2.) He showed that these figures representing numbers can be arranged and treated in sequence by a common generating principle, rotation of the diagonal of a quadrilateral, to unify an overall artistic or practical creation. 3.) He showed that the artist or practitioner retains some degree of freedom of choice when working within a system, since one can choose to add spaces to each other as is done when determining the length of the whole digit by adding the length of each individual phalange end to end, or one can return to the beginning point and incorporate only the most recent extension into the whole each time as is done in configuring the body proportions from head to toe.

How then do these geometrical constructions based on proportionally interrelated aspects of the circle, triangle and square figure in the design and transmission of a building plan from the mind of the designer to the necessary lines on the ground? For that I turn to the fifteenth century writings of Matthes Roriczer, Hans Schuttermeyer and Lorentz Lechler. Lon Shelby notes that "The medieval manuscripts by or about masons generally have a theme running through them that fundamentally connects the 'art of geometry' (*frey Kunst der Geometry*) to what Matthes Roriczer called 'drawn-out stonework' (*ausgezoge Steinwerk*) and what Hans Schuttermeyer, using the adjective 'measured' in the traditional hands-on sense, labeled simply as 'measured work'

(*Masswerk*)."[54] Referring to the sketchbook of Villard de Honnecourt, Shelby notes that problems of stereotomy were solved by medieval masons "primarily through the physical manipulation of geometrical forms by means of the instruments and tools available to the masons." [55] These, he says, were rule-of-thumb procedures, to be followed step by step and there were virtually no mathematical calculations involved. He calls this method constructive geometry, by which he means that technical problems were solved through the construction and physical manipulation of simple geometrical forms.

Erwin Panofsky observed in *Meaning in the Visual Arts* that what Honnecourt communicates through his sketchbook is an expeditive method in which the figure is no longer measured at all but is conceptualized rather as a system of straight lines superimposed on the work in progress that function as guiding lines rather than measuring lines.[56] It is important for our purposes to distinguish the function of guiding from that of measuring. Viewed in this way Honnecourt's system of lines in figure eighteen shares with the lines of Richard Tobin's reconstruction of the Canon of Polykleitos, the

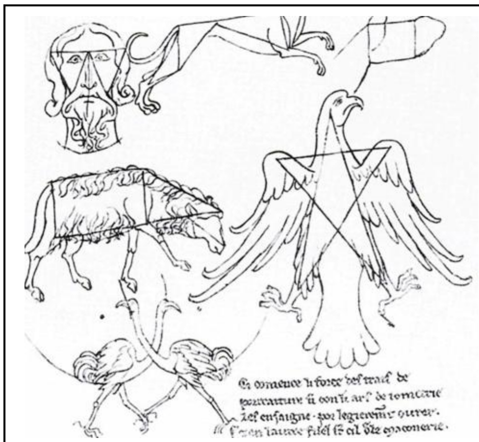

Fig. 26. From Villard de Honnecourt, Sketchbook, fol 18v. Paris Bibliotheque Nationale, MSF 19093.

[54] Lon Shelby, *Gothic Design Techniques*: *The Fifteenth Century Design Booklets of Matthes Roriczer and Hanns Schuttermeyer.* (Carbondale: Southern Illinois University Press, 1977). 53.

[55] Lon Shelby, "The Geometrical knowledge of Medieval Master Masons," *Speculum,* 47, no. 3 (July, 1972), 409.

[56] Erwin Panofsky, *Meaning in the Visual Arts: Papers in and on Art History,* (Garden City NY: Doubleday, 1957), 83.

function of guide lines, in Tobin's case, lines forming a connected series of rectangles that guided the sculptor in shaping the human figure that artistry 'extracts' from the stone block.

The essential idea of Shelby's "constructive geometry" was that Roriczer intended the 'right' form, meaning the proper form, was to be extracted out of the base form just as Poykleitos extracted the "living" human figure out of a static stone block. Matthes Roriczer puts it this way in the dedication of his 1486 publication *Büchlein von der Fialen Gerechtigkeit* (Booklet Concerning pinnacle Correctitude).

> I have tried...to explain the beginning of drawn-out stonework – how and in what manner it arises out of the fundamentals of geometry through manipulation of the dividers, and (how it) should be brought into the correct proportions – and to draw these hereafter-mentioned forms...[57]

In 1459 German master masons gathered in Regensburg and drafted an Ordinance creating a brotherhood of masons. They agreed to uphold the Points and Articles of the Ordinance that they set forth to govern themselves. One of these points prohibited anyone from doing the work who did not know how to "take the measure (*Mass*) or the extrapolation device (*Auszug*) out of the base plan (*Grund*)." The phrase 'to take the measure or the extrapolation device out of the base plan' is what is meant by extraction. It relates directly to the idea behind the Canon of Polykleitos, who derived or "extracted" a proportionally connected series of rectangles from the length of the distal phalange of the little finger. The proper proportions of the human body to be sculpted out of an amorphous block of Greek marble are "extracted" from the measure of this distal phalange by means of the extrapolation device. For Polykleitos the extrapolation device was the diagonal of a square or rectangle. To use the device it was rotated to create the

[57] Shelby. Gothic DesignTechniques, 83.

next step in the sequence, a new and longer 1 : √2 rectangle. In the method of Polykleitos as reconstructed by Tobin, we saw that it is entirely possible to develop a complex system using the single step of swinging the diagonal of a quadrilateral figure.

Shelby closed his monograph on the geometrical knowledge of the medieval master masons noting that the art of geometry for medieval masons meant "the ability to perceive design and building problems in terms of a few basic geometrical figures that could be manipulated through a series of carefully prescribed step to produce the points, lines and curves needed for the solution of their problems." He has an important admonition on how to successfully apply this observation.

> Since these problems ranged across the entire spectrum of the work of masons – stereotomy, statics, proportion, architectural design and drawing – the search by modern scholars for the geometrical canons of medieval architecture is appropriate enough, so long as we keep clearly in mind the kind of geometry that was actually used by the masons. The nature of that geometry suggests that these canons, when recovered, will not be universal laws which will at last provide *the key* to medieval architecture; rather they will be particular procedures used by particular masons at particular times and places.[58]

The geometrical canons will vary according to all manner of diverse circumstances, the mason involved, the geographic location of the practice, the period of history being investigated and the culture within which the practice occurred. The canon we are about to examine divides into procedures that are a continuously repeat a single step and those that involve a variety of steps used in a variety of sequence patterns, each combination designed to solve a specific kind of problem. We have looked closely at the first kind, exemplified by the Canon of Polykleitos. It consists of the repetition of the same step in a process that expands the base figure into the completed whole.

[58] Shelby. "Geometrical Knowledge". p. 421.

We look now at an example of the second kind of process for problem solving as it was published by Matthes Roriczer in his book entitled *Geometria Deutsch* (German Geometry). This example constitutes a sequential process in constructive geometry utilizing a variety of steps in varying ways to lead to the solution of a specific problem. Each new step is determined by information laid down in the previous step.

Shelby describes the difference between the presentation of two writers, one an arithmetical presentation and the other a geometrical presentation of the same problem. It is the problem of determining the length of a line equal to the circumference of a given circle presented in the Archimedean treatise *De mensura circuli*, (*On the Measurement of the Circle*,) translated from Arabic into Latin by Gerard of Cremona in the twelfth century. A fifteenth century treatise on geometry, *De inquisicione capacitatis figurarum* whose author is uncertain, gives an arithmetic solution as follows.

> Given the diameter, to find the circumference of a circle: Let it be that the circle is *a b* and the diameter of the circle is given as 14. Triple the diameter and it becomes 42. if you add to the product 1/7 of the said diameter, that is to say 2, there will be produced (the number) 44, which is the circumference of the circle. This is made clear by (theorem) 7 of the geometry of the three brothers.[59]

First, notice that the solution to the problem is entirely arithmetic, involving multiplication, division and addition. Second, it should be pointed out that the author chose a diameter that is easily divisible by the whole number two. Were the diameter to be fifteen or thirteen and a half, it becomes a more formidable problem in an age of severely limited calculating skills among the everyday workers in the vernacular world. Roriczer's geometrical solution involves no calculation whatsoever.

[59] Maximilian Curtze, " 'De inquisicione capacitatis, figurarum' Anonyme Abhandlung aus dem funfzehnten Jarhhundert," *Abhandlungen zur Geschichte der Mathamatik*, Heft VIII (1898). 37.

> He says If anyone wishes to make a circular line straight, so that the straight line and the circular are the same length: then make three circles next to one another and divide (the diameter of) the first circle in seven equal parts, with the letters designated h a b c d e f g. Then as far as it is from h to a, set a point behind (h) and mark an i there. Then as far as it is from i to k, equally as long in its circularity is the circular line of one of the three (circles) which stand next to each other as the figure stands made before.[60]

It is interesting that while Roriczer gives the means to find a solution to this problem, he does not spell out instructions for all of the steps. If a line segment can be geometrically divided into seven equal parts, it should be possible to arrive at a solution entirely by means of divider and straightedge or with the use of cord and pegs. The relatively simple geometric construction in figure twenty seven divides a line segment into any number of equal parts. A parallelogram is constructed over line AB consisting of seven arbitrarily equidistant intervals. To divide line segment AB into seven equal parts, first, construct a ray AC from point A. Then along the ray, from point A strike seven equally spaced arcs intersecting the ray and label the last intersection as C. Then set the compass point on B, and taking the distance to from B to C, place the point at A and strike an arc below line AB, the radius of which is the equal of BC. Placing the divider point at C, take the

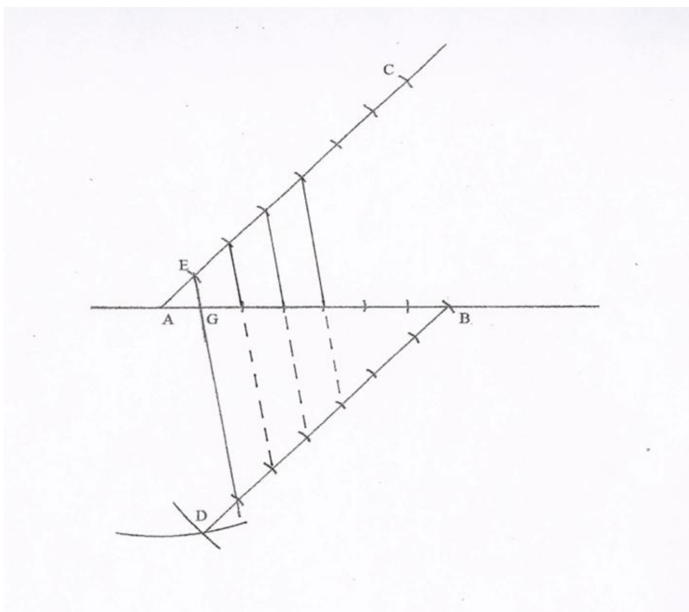

Fig. 27. Geometric construction to divide a line segment into any number of equal parts.

[60] Matthes Roriczer, *Geometria Deutsch*,. (Nürnberg: Heidelhoff, 1844) no. 9. Fol. 3^{v}. fig. 32. Translated with illustrations in Shelby, *Gothic Design Techniques,* 120-121.

distance CA and setting the point on B, strike an intersecting arc and label it D. Then copy the series of arcs along AC onto line DB. Beginning with the arc at E and its corresponding arc on DB, connect each pair of arcs along AC and DB. Where these seven lines intersect on line AB, mark the seven equally spaced divisions of line segment AB.

Knowing how to divide a line segment into seven equal parts, we are prepared to look at the problem in Rorizcer's *Geometria Deutsch.* The task is to construct a straight line whose length is equal to the circumference of a given circle. An ancient approximation of the circumference is that it is equal to three times the diameter plus less than 3 $^{1}/_{7}$ and more than 3 $^{70}/_{71}$. Roriczer's geometric solution will set out a distance of 3 $^{1}/_{7}$ times the diameter.

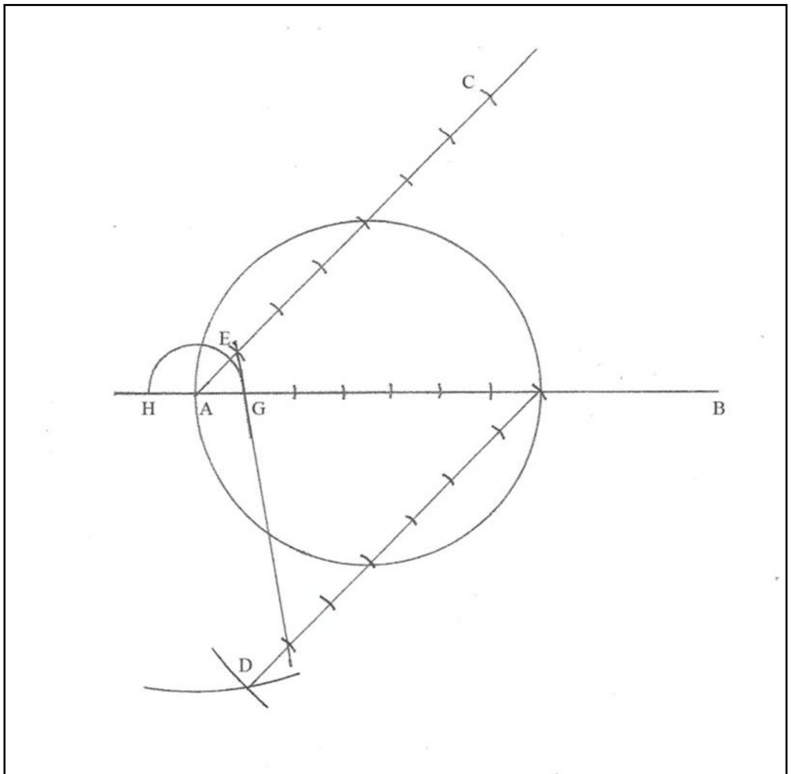

Fig. 28. Roriczer's geometric solution for a straight line equal to the circumference of a circle

His first instruction is to construct three adjacent circles of equal size. In Figure twenty eight since two of the circles are not used other than to add their diameters to the length of line AB that serves as the diameter of the three circles, in the interests of saving space the two additional circles have been omitted in this drawing. In the full drawing point B at the end of line AB would be at the right edge of the third circle.

The steps needed for the solution are simply to divide the diameter of the circle into seven parts, and then to add one of those parts to line AB, remembering that it is the combined diameters of the three circles. How to divide a line segment into equal parts we now understand and so we do it here to the segment of line AB that lies within the left circle and constitutes its diameter. All steps can be carried out with equal ease using cord and pegs at a construction site, and the greatly expanded scale at the construction site significantly reduces the impact of error in the laying out process. Roriczer offered a prescriptive solution for which there is no proof given but that is sufficiently accurate to be useful, as otherwise it would not have become a part of common practice.

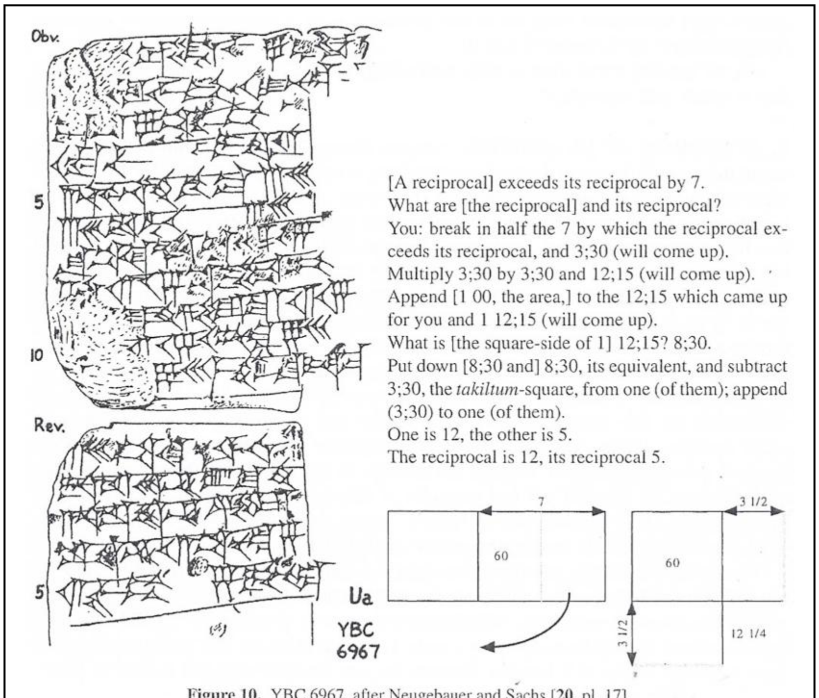

Fig. 29. Eleanor Robson's solution by constructional geometry of the mathematical problem on the cuneiform tablet Plimpton 322.

The example just seen uses a geometric construction to arrive at a solution. There is another aspect of the practice that geometrically creates a figure and then manipulates elements of the figure to solve the problem. An example of this kind of practice comes from the *Booklet on Pinnacle Correctitude.* A precedent from the Ancient World for this kind of problem solving is the Babylonian problem on tablet YBC6967 seen in figure twenty nine, to identify two reciprocal numbers, one of which exceeds the other by seven. Eleanor Robson hypothesized that geometric figures are manipulated for the solution of the problem. Because the product of any two reciprocal numbers is 1, or any power of 60 in Babylonian base six mathematics, she conceptualized the problem as the task of finding the length and width of a rectangle whose area is sixty. She arrived at the solution by breaking the rectangle into appropriate quadrilateral figures and then by shifting these figures around a square that is an integral part of the original rectangle whose area is sixty.[61]

In Roriczer's case we will find Shelby's constructional geometry applied to determining the position and size of architectural elements rather than to finding the solution to a problem of numerical relationships. Returning to Roriczer's book on pinnacle correctitude, the pinnacle is a relatively smaller vertically oriented decorative detail appearing in the context of a cathedral facade or roofline, though it can appear on heavy furniture and ornamental boxes of metalwork as well. It connotes the idea of upward aspiration and consequently is generally applied at roof level and on towers. Relatively smaller in size, its base figure from which the position and size of its lines are to be extracted could be drawn out on the tracing room floor. From the drawing on the tracing room floor by means of the extraction device the full pinnacle figure could then developed with the base figure serving as the template from which the measurements are applied to the stone piece being worked into shape. We will not follow through the entire process for extracting a pinnacle, as it is lengthy. Rather we will examine the process far enough to see clearly how the base figure is developed and how the extraction of the

[61] Eleanor Robson, "Words and Pictures: New Light on Plimpton 322". *The American Mathematica Monthly*, 109, No. 2, (Feb. 2002). 115-116.

elevation from the base figure works and then we will very briefly look at the process for design of a gablet, noting that essentially the same kind of base figure is developed, but with a degree of variation from the base figure for the pinnacle process.

Roriczer's illustrations for the process of pinnacle and of gablet design are taken from their reproductions in Shelby's book on techniques of gothic design.[62] Using the

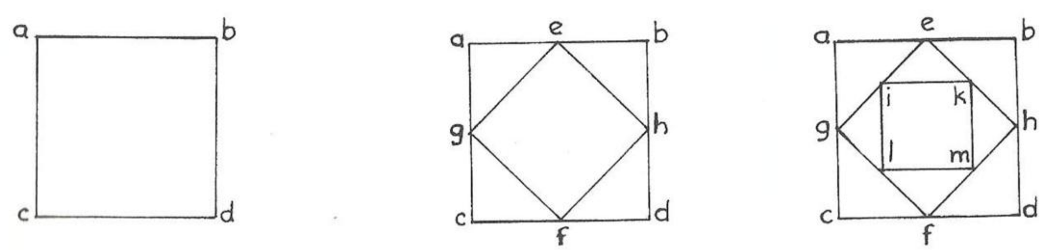

Fig. 30. Roriczer's first three steps construct two related squares within the original square. From Roriczer's *Büchlein der Fialen Gerechtigkeit.*

extraction device (*Auszug*), the task is to extract the elevation of a pinnacle from the base figure (*Grund).* First the base figure must be drawn. Again Roriczer's booklet tells us what to do, but not precisely how to do it. The base figure is essentially a set of three concentric squares that are proportionally related, familiar to us from the figures of Tons Brunés. Roriczer's instructions shows the correct form for three concentric squares in the proper proportion to each other as seen in the three drawings in figure thirty. There are a number of simple techniques for geometrically constructing a square, all easily carried out with cord and pegs. Points (g), (e), (h) and (f) in the second diagram are all found by using the simple construction for finding the mid-point of a line segment. The four points are connected on the diagonal to construct the middle square, and similarly for points (i), (k), (m) and (l), at the center of lines (ge), (eh), (hf) and (fg). In the next figures the sides

[62] Shelby reproduces the figures for pinnacle design from Roriczer's *Büchlein von der Fialen Gerechtigkeit* on pages 4 through 106 of *Gothic Design Techniques* and those for the design of gablets from his translation of the *Wimperbüchlein* on pages 107 through 111 of the same book.

of all three squares are shown in parallel as in figure thirty one. But Roriczer does not tell his readers how to construct the three squares in parallel.

It is reasonable to think that any mason authorized to extract the elevation out of the ground will know how to do this, but we are left to work this out on our own. The

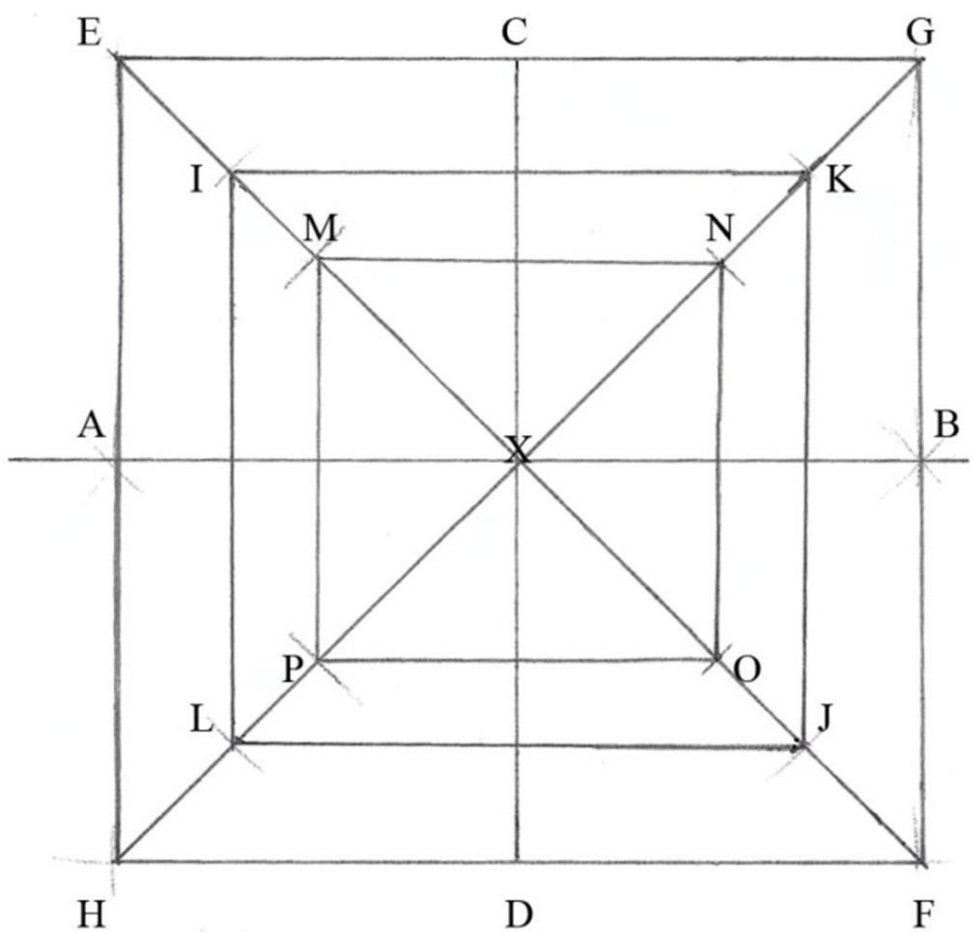

Fig. 31. How to draw the squares in parallel with each other. (Drawing by the author)

steps involve simple geometry and the process is shown in Figure thirty one as follows. Let X be the center point of the entire construct. First lay out line AB. Then using the appropriate geometric construction, erect a perpendicular line CD at X, the mid-point of AB. and let it be longer in length than line AB. From point X Copy AX onto the vertical centerline above and below X to establish point C and point D. With the divider copy AX and with A as the pivot point strike an arc above A through what will become point E. Do the same from point C to establish point E at the intersection of the two arcs. Do the same from C and B, B and D and D and A, establishing points at G, F and H. Connect points E, G, F and H to create the outer square. Now draw the diagonals connecting HG, and EF through center point X. Copy XA onto EX and onto GX, FX and HX, marking points I, K, J and L. This is no different than constructing a circle whose radius is XA. Connecting these four points constructs the middle square. To construct the inner square, mark only the intersection point respectively of lines from A to C, C to B, B to D and D to A with diagonals EF and HG to establish points M, N O and P. Connect these points to construct the inner square.

The remaining steps to construct the base figure for the erection of Roriczer's pinnacle use the points already established to mark out additional points on the base figure. In the process Roriczer demonstrates what to do, but again leaves out specific instructions for a crucial step. Figure thirty two shows the completed base figure as drawn by the author. Eight short lines are equally placed at O between

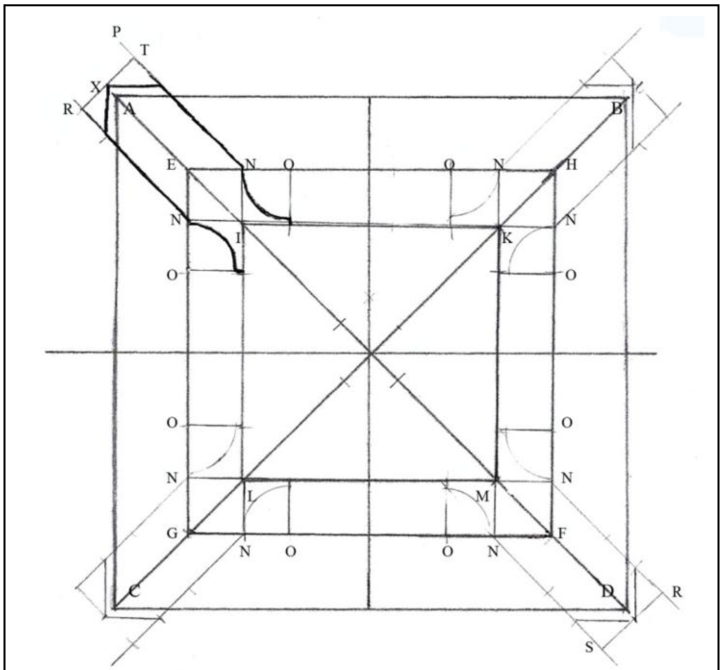

Fig. 32. The completed base figure from which the extrapolation device will extract pinnacle as the elevation.

the inner and the middle squares. Roriczer's implicit instruction for this is to take one third of the distance from I to N, meaning, horizontally along line IK. It was shown in figure nineteen how to divide a line segment into (n), that is, into any number of parts but this does result in a complicated drawing overall. However the line segment in the present drawing from the upper left corner A of the outer square to the that of the middle square at E is in fact equal to one third of the line segment IN and may simply be copied into the drawing for each of the eight lines marked O. One can only speculate as to whether this indicates Roriczer intended his book to be used only as a booklet of reminders for skilled masons who would know that that was the case from past experience with similar devices, or whether this simply represents an oversight by an author so familiar with the process that it is overlooked in writing out the explanatory text.

What remains is the process of extracting the elevation out of the base figure using the extrapolation devise (*Auszug*), seen in figure thirty three. The extrapolation device is an extension of the vertical center line of the base figure that is constructed by taking length AB from the base figure and marking it off six times along the extension of the center line. This sets the length of the section of the pinnacle termed the body of the pinnacle. The width of the pinnacle body at various points will be determined by copying lengths off the base figure. The pinnacle consists of three parts, the body, the cap and the finial. The body is the main shaft of the pinnacle. The cap is a tall steeply tapered section coming nearly to a point. The finial is a

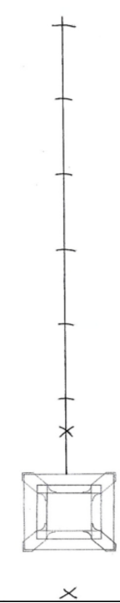

Fig. 33. The extraction device.

decorative piece placed on top of the cap. The extrapolation device is identically used for designing both the body and the cap. The finial is laid out using the same device in essentially the same manner.

As seen in figure thirty four, the first step is to construct horizontal lines at top and bottom along which the dimensions across the body will be set. The bottom line is already there as the AB line of the base figure. The top line is at the arc marking VI. Both top and bottom lines are a little wider than the base figure, the top marked XRY and the bottom TSV. All of the necessary dimensions of the pinnacle body will now be copied off the base figure and marked onto the top and bottom lines at (a), (e), (g), (k), (l,) (h), (f), and (b), and then vertical lines are drawn to connect pairs of points top and bottom. The lower points however, are placed along diagonal lines at (e), (g), (k), (l), (h) and (f). Because the process is so repetitive we examine only the placement of the lines for the body of the pinnacle.

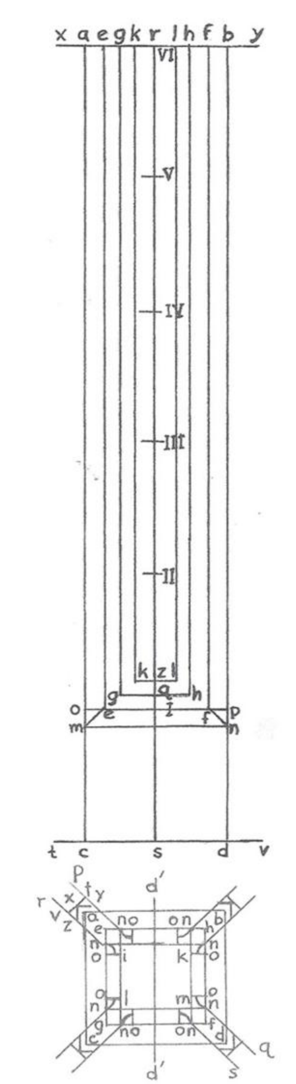

Fig. 34. Dimensions copied from the base figure onto lines TSV and XRY set widths that extract the pinnacle.

While providing reminders to the knowledgeable mason, he has not indiscriminately given away the ability to copy the process, perhaps a concession to the Regensburg protocol. We noted earlier with figure thirty one that Roriczer left the parallel orientation of the squares composing the base figure unexplained. While the text for the steps spells out the order in which they must come and the illustrations show what they must look like, he gave no instructions for making the concentric squares in the parallel orientation of the base figure.

Shelby warns that Roriczer's booklet on gablets the *Wimpbergbüchlein,* is different from the *Fialenbüchlein* in that the latter is all but self-explanatory, while the former definitely is not so. He points to the difference as follows.

> It also points to a basic difference between the two booklets, for R. attempted to make the pinnacle booklet completely self-explanatory, whereas both text and illustrations of the gablet booklet were highly condensed and not fundamentally self-explanatory. One must not only presuppose a knowledge of the quadratic technique as set forth in the *Fialenbüchlein* but be willing to puzzle the application of the technique to the more complex form of the gablet, without the aid of full verbal descriptions and step-by-step illustrations.[63]

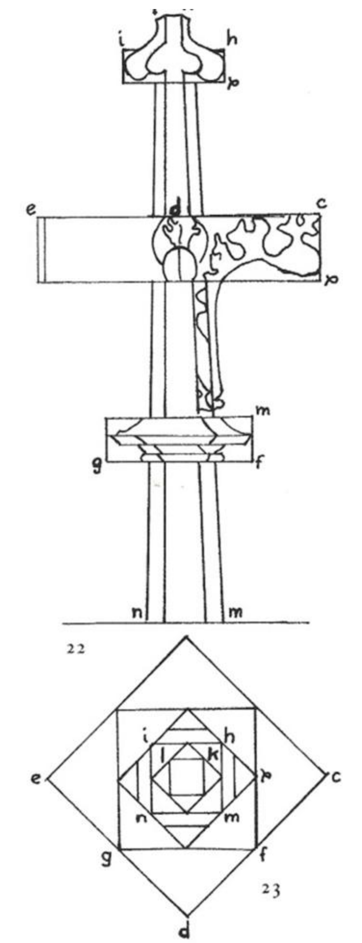

Fig. 35. The Gablet base figure and the extrapolation device.

The base figure works in relation to the extraction device in exactly the same way. One first constructs the base figure and then, placing it below the extraction device, the dimensions are copied from the base figure onto the extraction device to design the gablet. Note in figure thirty five however, that while the three outermost concentric squares are in the same orientation as for pinnacle design, there now an additional three squares in square (inmh) and a further square only partially contained within (inmh). The process of extracting the elevation

[63] Shelby, *Gothic Design Techniques,* note 1, page 178.

from the base figure was a generally applied procedure that could be varied in its specifics to suit different design tasks. A master mason knew the general procedure and was knowledgeable about the variations, techniques that were traded between master masons.

Three qualities contribute significantly to plan-net technique; 1.*dynamis* in system transitions, 2. interrelational properties of basic geometric figures and 3. sequential proportionality. The phenomenon of *Dynamis* is apparent in Plato's *Meno* dialogue as Socrates used the proportional relationship between elements of geometrical figures to solve a problem in organization of space. Tons Brunés demonstrated the inter-relatedness of the circle, triangle and square, functioning in combination as a critical factor in problem solving by manipulation of geometric figures. Richard Tobin's reconstruction of Polykleitos' *Kanon* used sequential proportionality to extract proportions for a complete human form from an initial square base figure. In the late fifteenth century Matthes Roriczer put the use of these characteristics into published form. It remains to examine the practical cord and peg techniques needed to carry out such procedures without the use of modular measurement.

The steps taken for construction of a figure drawn on paper or tracing floor with divider and straightedge and the steps used to draw ground line at the construction site with cord and pegs are exactly the same. The *Sulva Sutras* of Vedic India show how cord and peg techniques were used in non-literate construction projects. Sutra techniques of geometry and modular measurement demonstrate the cord functioning both as a template to convey measurement and as a tool to construct a geometric figure. In 1875 G. Thibaut published an article on early Indian writings known as the Ṡulvasūtra-s in the *Journal of* *the Asiatic Society of Bengal.* It was re-published in India as *Mathematics in the Making in Ancient India.* A sutra is a set of instructions and the Ṡulvasūtra-s are the "sūtra-s of the cord." They give practical rules to the Sūtra technicians for the use of cord and pegs to lay-out the ground lines upon which to construct altars of fired bricks.

The altar's purpose was to serve as the correctly shaped location to send a fire offering heavenward to gain specific ends from deity in a universe the Vedic people thought to be square. The altars were constructed in various shapes associated with gaining a specific kind of favorable result. Because the altar was the ritual site it was as perfectly shaped as possible and because the universe was square the altar was aligned with the universe by constructing it in conceptually square units.

Every altar was seven and one half square puruṣa-s, a unit of length, and was built as five layers, each containing two hundred bricks totaling one thousand bricks. The body of the falcon altar was comprised of four square puruṣa-s, three and a half purusa-s for the two wings and tail and the remaining half puruṣa was subdivided into squares of five measured units to a side and was used to more perfectly shape the altar as a falcon. [64]

The variety of shaped altars and the ritual restriction to seven and one half purusa-s required 'rules of the cord' that enabled technicians to lay out a square that was, for example, equal to two or more smaller squares or a square equal to the difference between two squares. Technicians had to make squares and rectangles exactly rectilinear and when squares were transformations of triangles and circles, all such equivalent figures had to be exactly equivalent. The many Ṡulvasūtra-s are sets of rules to turn rectangles into squares, squares into rectangles and to construct triangles and circles

[64] G. Thibaut, *Mathematics in the Making in Ancient India,* ed. with introduction by Debiprasad Chattopadhyaya, (Calcutta: K.P. Bagchi and Company, 1984). 7-8.

equal in area to a given square or rectangle. No proof is involved. The sutras say simply that to lay out this particular thing, do the following. Sutra technicians were interested in geometrical truths as rules that enabled them to exactly control the altar construction process.[65]

Using cord and peg methods the non-literate world solved problems of astonishing complexity and sophistication and they were favored in matters of lay-out, especially at the scale of complex altars, large buildings and fields. The larger the scale of the project, the greater the opportunity that existed for accumulative error in repeatedly moving a rigid measuring rod across a site, a problem not eliminated in construction until the invention of the retractable measuring tape in the 1850s. As essential as cord and pegs were in the lay-out at the altar construction site or in field measurement, cords made of organic material nevertheless stretched. For this reason among the Egyptians there was a class of specialists, the *harpedonaptai*, or rope-stretchers and in India this specialization fell to the sutra technicians, "the *Śulbavid* "the person who was expert in the science of measurement . . . the expert in measuring, uniform stretcher of the rope . . . ." [66]

The exacting nature of ritual work is clear from the Babylonian cuneiform texts and from the Vedic Ṡulvasūtra-s. In Old Babylonia the foundation of a subsequent temple had to be placed exactly on the foundation of a preceding temple. Nabonidus, the last king of Babylon (555-538 BCE), closes a text with a frequently repeated formulaic phrase;

> I searched for its old foundation; I dug down eighteen cubits into the ground . . . I laid its brickwork on the foundation of Naram-Sin, son of Sargon, *not protruding or receding an inch*.[67] (italics added)

[65] G. Thibaut, *Mathematics in the Making* 7-8.
[66] D.P. Kularia, *Kātyāna Śulbasūtra, with Commentary,* (New Delhi: Devesh Publications, 2009). vii.
[67] Richard S. Ellis. *Foundation Deposits*, 183.

Regarding the Ṡulvasūtra-s, Thibaut describes this attention to accuracy saying

> The Ṡulvasūtra-s, however, introduce us to a different technological climate...The bricks are deliberately made according to certain specific shapes and sizes; these are deliberately dried and burnt. Attempt is made to determine exactly how much in size and area these lose as a result of drying and firing, so that provision may be kept in the original bricks for their shrinkage and eventually burnt bricks are obtained according to their exact area required for the altar-making.[68]

In the same exacting vein, the Ṡulvasūtra-s offer precise instructions on how to lay out an exact geometric figure with cord and pegs.

Two examples of sūtra instructions follow here, one a simple geometric problem involving squares, and the other a more complex process of laying out a simple altar. In figure thirty six the first problem is that of deducting a smaller square from a larger square. The sūtra says, "If you wish to deduct one square from another, cut off from the larger one an oblong with (having) the same size as the side of the smaller one; draw (rotate) one of the sides of that oblong across to the other side; where it touches the other side, that piece cut off; by it the deduction is made." Given that (*abcd*) is the larger square as illustrated in figure thirty six, a rectangle whose width (*ce*) is equal to the small square to be subtracted, is constructed within the large square. The line (*ef*) is rotated to the right, touching (*bc*) at(*g*). A horizontal line at (*g*) marks off a new and

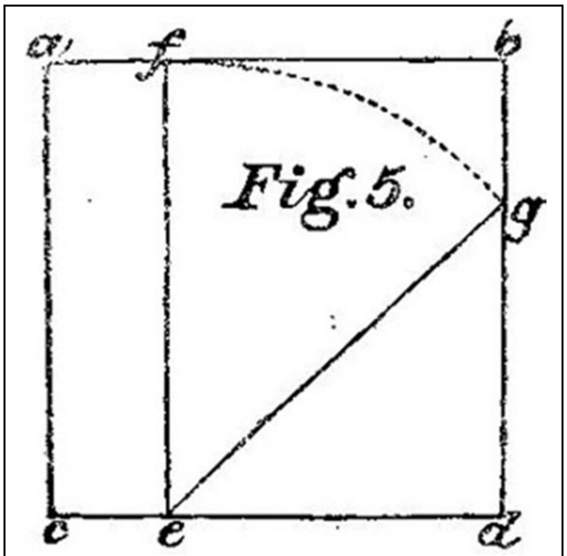

fig. 36. Vedic Indian cord and peg solution to subtracting one rectangle from another. Drawing from G. Thibaut.

[68] G Thibaut. *Mathematics in the Making,* xvi-xvii.

smaller square, the area of which is equal to the difference of the two given squares, that is, the area of the new square is equal to the area of the larger square less the smaller square that was subtracted away. Consistent with the ritual's requirement regarding the square, the technicians conceptualized the problem in squares, achieving relationships by manipulating geometrical figures to create one square subtracted from the original.

The next case uses numbers but does not calculate the solution by numbers. Rather, it uses the numbers to serve as templates affixed to the cord. Most projects would never have been carried out entirely without numbers. Measured points were marked on the cord, perhaps in the same way that yard goods are measured out by sales clerks passing the fabric from hand to hand over a fixed ruler on the table. Modular measurement is used here in a way that does not bring accuracy into question through repetition. We will see in a later chapter that smaller measurements could be used in plan-net development of a floor plan in much the same way to locate and size small details of the finished project such as windows and doors and placement of wall cabinets once the overall larger structure was marked out. Measurement by number was also practical to lay out the small scale initial figure from which the whole plan-net was extracted.

This sūtra lays out the ground lines for a shaped altar. The first task was to lay out the *prācī,* a thirty six foot east-west line positioned according to astronomical considerations. Ritual required the east and west side of the altar to be at right angles to the *prācī*, the west side thirty units long and the east side twenty four units long. The first step was to mark the cord at thirty six feet from the cord end that served as the western end of the cord. This is what was referred to above as use of the marked cord as a template. Then a mark was also placed at an additional eighteen units at the eastern end beyond the thirty six foot mark, extending the length of the whole to fifty four units on the cord. From the western end of the *prācī* a mark is placed at twelve units and a second mark is placed at fifteen units. Since ritual accuracy was crucial to a successful sacrifice, the key points along the cord used to stake out the perimeter are measured out along the cord under carefully controlled conditions, as mentioned above, best done perhaps in the manner of a modern sales clerk measuring yard goods.

In figure thirty seven the first step in deploying the cord is to form a triangle whose base is the thirty six unit *prācī* line, the side is a fifteen unit line and the hypotenuse is the thirty nine unit diagonal. The diagonal of a fifteen by thirty six unit rectangle is thirty nine units. Since $a^2 + b^2 = c^2$ then $\sqrt{(225 + 129)} = 39.127$, the margin of error being a little over one tenth of a foot. The angle opposite the hypotenuse is a right angle, guaranteeing that the line for the west side is at right angles to the *prācī* line. This process is repeated on the north side fixing the northwest corner of the altar at (d). The distance between the northwest and the southwest corner stakes is fifteen plus fifteen units, giving the required thirty feet for the west side. To fix the southeast and northeast corners the process is reversed in direction and carried out in the same way at (e) and (f) but with this difference. Once the cord is stretched taut

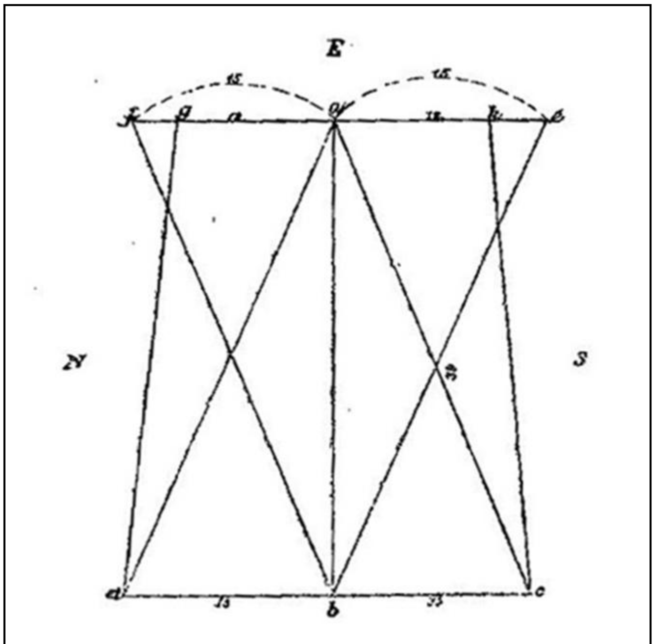

Fig. 37. Cord and peg lay-out of an altar according to the Ṡulvasūtra-s. From G. Thibault.

and the fifteen unit mark staked, a stake is placed at the twelve unit mark to fix the actual location of the east corners.[69] Twelve plus twelve gives an east side length of twenty four feet.

Again, there are no proofs in the sūtra directions and emphasis is on the ritual accuracy common to the Babylonians, the Egyptians and the Vedic Indians. In the Ṡulvasūtra-s we see problem solving that involves measurement by placing marks on the cord, but requires no calculation, the measurements being applied to a cord that will function as a template. The Ṡulvasūtra-s display concretely the solution of construction problems by manipulation of geometrical form using cord and pegs rather than by manipulation of numbers using calculating techniques.

Manipulating interrelated forms of the circle, triangle and square marked out a space unified in its external proportions and internal divisions as a single system. An equally unified system of sequentially proportional steps enabled transmission of construction information in a non-literate context. This chapter began with an observation of the ancient Greeks called *dynamis,* that when systems change, some elements remain the same as other change. Four factors examined are the proportional interrelationships of the circle, triangle and square demonstrated by Brunés, the reconstructed methodology of the Canon of Polykleitos by Tobin, Shelby's translation and description of applications of proportional relationships as developed by Matthes Roriczer in his booklet on the rectitude of pinnacles and gablets, and finally the practical methods of the use of cord and pegs demonstrated in the Sulva Sutras, translated and published by Thibaut.

[69] G. Thibaut, *Mathematics in the Making,* 12-14. There are a number of commentaries on the Ṡulvasūtra-s. In this example Thibaut is translating the Baudhāyana commentary. D.P. Kularia translates the Kātyayāna Ṡulvasūtra. Thibaut gives examples from a variety of commentaries. The sūtra-s are precise but sparsely worded, and not always easy to follow. Thibaut's diagrams and their explanations are clearer than those of Kularia.

All of these are factors that contribute to the plausibility of plan-net methodology in a non-literate context of architectural problem solving. This begs the question examined in chapter four; What is the evidence for the concept of a plan-net, and the evidence that such a method such had continuous life until problem-solving by geometrical manipulation was replaced by manipulation of numbers?

Chapter IV

The Plan-net Narrative over Time

The term plan-net is adopted for use in this study from an Egyptian hieroglyphic word found in texts inscribed into the foundation stones of certain Egyptian temples, the meaning of which is described by Alexander Badawy in *Ancient Egyptian Architectural Design*; *A study of the Harmonic System.* Though his analyses follow the method of those who sought the source of beauty and harmony in architecture by overlaying favored geometrical figures on floor plans seeking points of coincidences between them, his discussion of the foundation texts cannot be overlooked. They constitute the earliest available reference to the actual form taken by the system of analysis proposed here, that of a rectilinear network of parallel lines. The Egyptian foundation texts document the methodological idea and describe the ritual participants and the ritual objects, but give little evidence for full process itself. Substantial documentation for an actual method does not appear until Matthes Roriczer's publication on pinnacles and gablets in the fifteenth century AD. The present chapter briefly describes the method of plan-net analysis, examines iconographic evidence supportive of such a process and similar evidence that the tools and concepts were gradually symbolized in a transition from elements of practice to elements of ritual.

In chapter three Brunés and Roriczer demonstrated the broad applications possible for the square in design and execution processes. The Canon of Polykleitos demonstrated the importance of the diagonal in carrying out a proportionally sequential process. The Sulva Sutras provided instruction on cord and peg methods. Rotation of the diagonal of a quadrilateral figure is the fundamental step in the plan-net process and sequential proportionality results from the process itself. In the following I use the term quadrilateral referring to both squares and rectangles and meaning to exclude non-rectilinear figures. Every quadrilateral figure has two diagonal lines, each longer than any side of the figure. The diagonal of the square stands in ratio to the side as √2 : 1, that is 1.412 : 1. When the diagonal is rotated from a corner pivot point to overlay one of the sides, it extends that side interval by .412. If we consider that side as a segment of a line extending in both

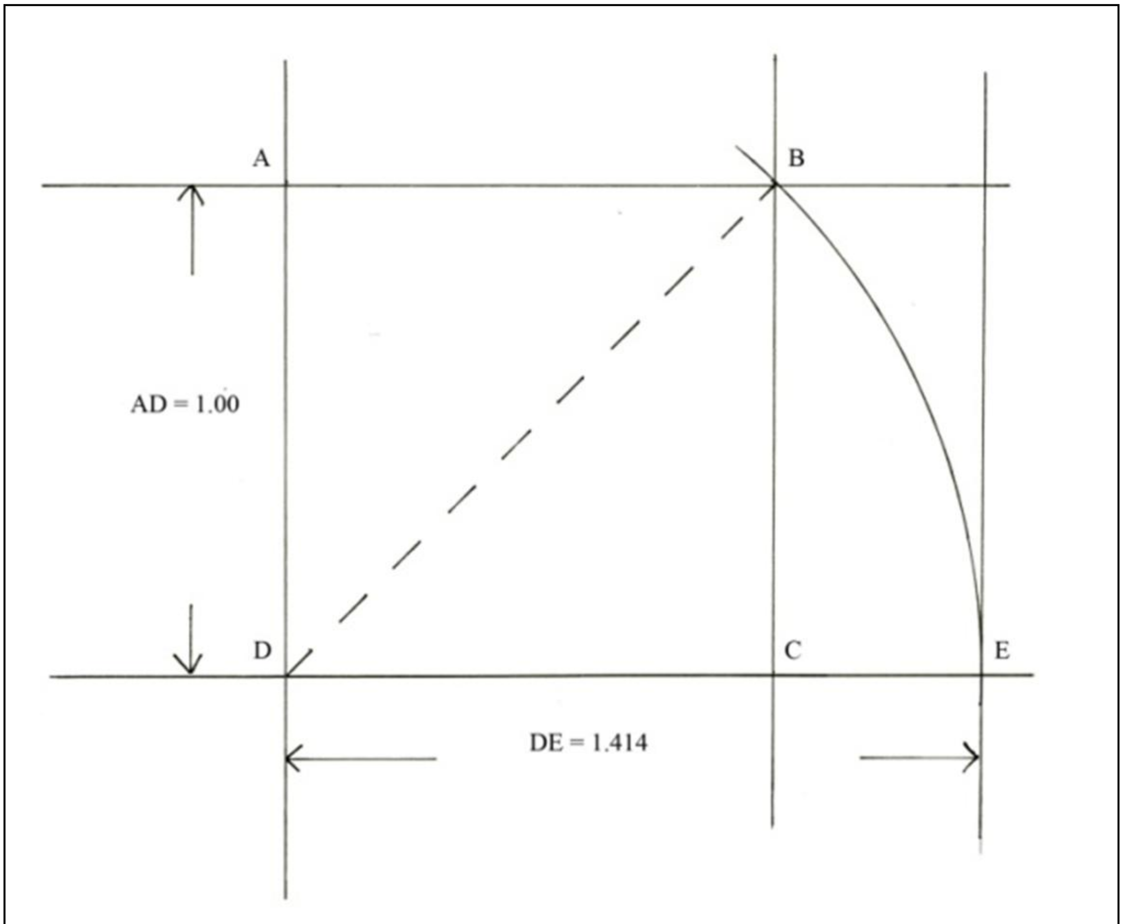

Fig. 38. The diagonal of the square rotated to form a 1 : √2 rectangle

directions as far as desired, as for example DC in figure thirty eight, we can mark the now rotated distal end point of the diagonal onto the extended line DC at E. Because there are two diagonals the same also can be done on the other parallel line AB thus establishing a point on each of the two parallel lines. These two points define a new line at E perpendicular to AB and DC.

In subsequent steps as shown in figure thirty nine the two diagonals of any previously formed quadrilateral figure may be rotated in the same manner to intersect in any direction with a pair of parallel lines, creating create a new and yet larger

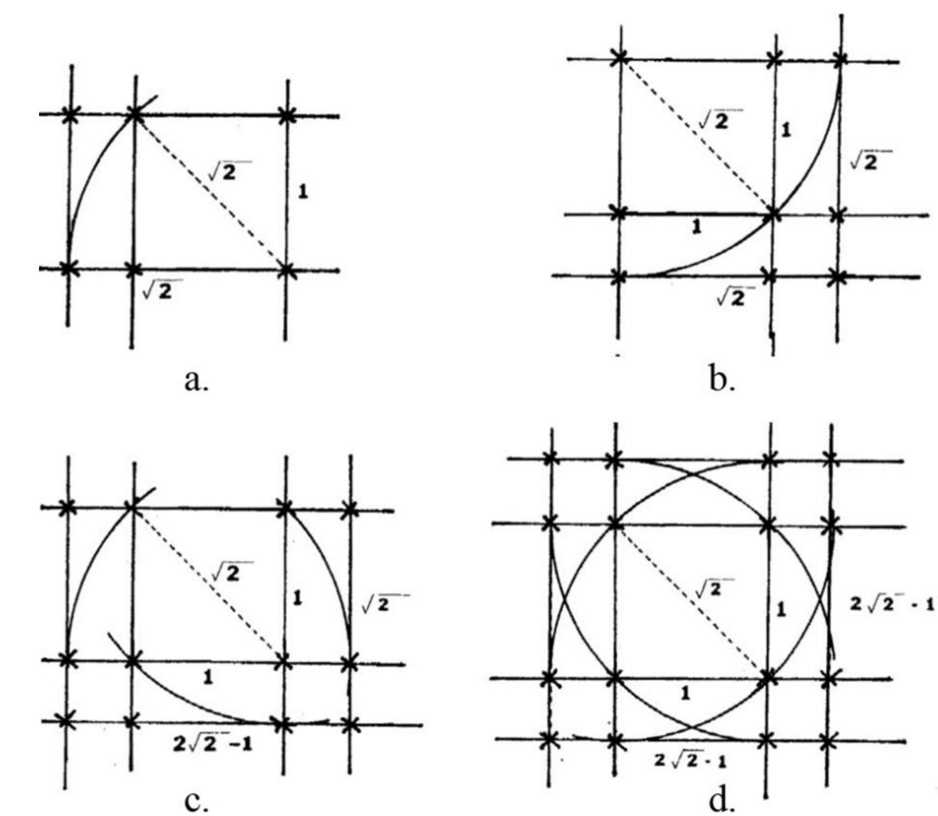

Fig. 39. The diagonal of the square rotated in various directions creating expanded figures according to need.

quadrilateral. In Figure 39a the diagonal from lower right to upper left swung to the lower parallel line fixes one of the two points for the √2 interval forming the rectangle. In Figure 39b the original square is enlarged by swinging the single diagonal in two directions from the same pivot point. 39c and 39d show the diversity of forms that may be derived from a single square.

Expanding the plan-net by adding new lines to it results in a rapidly growing cascade of new intersection points. While the initial square is defined by four nodes, there are six nodes in Figure 39a, 39b has nine nodes, 39c has twelve and 39d has sixteen nodes. These intersection points are nodes available to the designer for new steps and in practice their selection will be decided based on designer intent. Any selected node is a point on which either a leg of the divider can be set or a peg can be driven to which a cord may be attached. The other leg of the divider or the distal end of the cord are free to rotate either left or right from the node of the diagonal opposite the pivot point. The point at which they intersect an existing line in the plan-net is marked on that line as one of the two points that will establish the next line to create a new and larger quadrilateral figure. This is the process that generates new space to be included in the developing plan, and of this process one may say as does Matthes Roriczer that the plan is being extracted from the base figure, in this case the initial square.

The connection between this network of lines and the expression in the foundation texts of "casting the plan-net on the ground" is obvious. Badawy noted that Egyptian monumental buildings sometimes came into being as harmonious whole units and sometimes also by an accretion process across centuries. Where accretion over time results in a harmoniously unified structure, it indicates these buildings were initially constructed and subsequently added to according to carefully archived governing rules and specifications.[70] The Egyptian practice of referring to archived building records and specifications is well documented.

Badawy noted also the intense interest of the Pharaoh in monumental architecture constructed under his watch. This speaks against the Pharaoh leaving execution of details to those in positions of lesser knowledge and skill. One text says that “King Sahure (fifth dynasty) followed the progress of the work on two stelae for the audience hall ‘to be done in the presence of the king himself,’ and that ‘every day’ his majesty had ‘color’ put on them ‘and had them painted in blue.’ Several New Kingdom records describe the interest of King Thutmosis III in foundation ceremonies.

> My majesty ordered that the foundation ceremony should be prepared (at the approach of) the day of the Feast of the New Moon, to spread out the plan-net upon this monument....This god (here meaning the Pharaoh) assumed the station (for) the spreading out of the plan-net...Behold the majesty of this revered god desired to do the extending of the cord himself. [71]

Given such interest in details and that the Pharaoh extended the cord himself, he was unlikely to turn over details to subordinates without continuing in direct supervision of the work.

Greco-Roman foundation texts from Egypt describe specific foundation ceremonies associated with laying out ground lines. A text from the birth-house of Ermani, now destroyed, refers to the house of Rattaoui, the *mamisi* of Horus, “that was loosened by means of the plan-net, that was measured perfectly”.[72] According to Badawy the verb translated as “loosened” can also mean “to unfurl” or more abstractly, “to solve.” Badawy did not limit the phrase to stretching a cord to mark out just the axis or outline of

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[70] Alexander Badawy, *Anchent Egyptian Architectural Design*: A *Study of the Harmonic System*, (Berkely: University of California Press, 1963), 6-8.
[71] Badawy, *Egyptian Architectural Design*, 11.
[72] F. Daumas, *Les Mammisis des Temples Egyptiens*, (Paris: Soc. d’ed.”Les belles Lettres,” 1958), 342.

a building. In figure 40 the hieroglyphic figures of an inscription referring to Thutmosis III in the ritual act of "loosening of the plan-net" show the Pharaoh performing in person a double ceremony, each of the phases being described individually and in differing language. "The king himself, who performed with his two hands the stretching of the cord and the spreading out of the plan-net, putting it on the ground."[73]

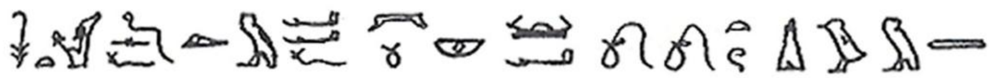

Fig. 40."The king himself, who performed with his two hands the stretching of the cord and the spreading out of the plan-net, putting it on the ground."

Badawy considered the first phase of the process, described on the left side of the inscription, to be extension of the cord to plot the axis of the building or to mark its outline. The second phase, inscribed on the right side, is to stake out the ground lines using the special plan-net *wꜣwt*. Given the interdependence of ground lines in marking internal divisions and perimeter, the purpose of the first step seems more likely to be to plot the axis line than to mark the outline. *wꜣwt*, the word for plan-net, is etymologically related, he says, to *wꜣwꜣ*, meaning to 'plan' or 'to project' and to *wꜣt*, meaning 'cord'. He continued to say that the scene described by this text usually features the king facing Seshat, the goddess of architecture and reckoning, both driving a tall stake in the ground. A cord is wound around both stakes as a symbol for the plan-net. Perhaps the simplest way to account for the two phrases of the inscription lies in the fact that stretching the cord is implicitly the means of putting the plan-net on the ground.

As seen in figure forty one some texts define the striking of the *nbi* stake. "He spread out the plan-net, the *nbi* stake, being in his hand" Other similar texts help draw an

[73] Kurt Sethe, *Urkunden des 18 Dynastie, I,* (Leipzig: 1906,) Also J.H. Breasted, *Ancient records of Egypt*, II, (Chicago, 1902), 152.

exact picture of the process, where the length of the building seems to be a fixed dimension, while the width is determined in proportion to the length according to a rule.

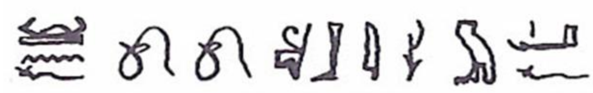

Fig. 41. "He spread out the plan-net, the *nbi* stake being in his hand."

"Its length is perfect, its width conforming to the need (*ḏʒr*), its height exact." This interpretation is corroborated by a text from Dendera which states it explicitly: "Its length is exact, its width according to the formula (*ḏs*, *ḏʒjs*), its norm is in excellent work."[74]

Badawy went on to suggest that the abstract concept of 'loosening' or'solving' implied in the term *wḥ'* makes sense if the Pharaoh and the priest set out the outline of the building and then other technically skilled people fill in the inner divisions later from drawn plans. He proposed that the plan-net was loosened or solved by using the stake and cord to lay out a completely closed rectilinear figure on the ground to outline the building. I propose another possibility in which the plan-net procedure is a generative process that extracts new space from an initial base figure by manipulating the elements of that figure. The net of rectilinear lines produced by casting the plan-net on the ground served to place all of the interior divisions and significant architectural figures as well as the perimeter lines of the building.

The plan-net process just demonstrated can be applied to an ancient Egyptian floor plan drawn on an ostrakon. An ostrakon is a broken piece of pottery upon which a written text or drawing is placed. An ostrakon from the Valley of the Kings at Thebes and now in the Cairo Museum contains a floor plan drawing done in red ink. It is for a small rectangular building whose roof is to be supported by four columns. The door is drawn in

[74] Badawy, *Egyptian Architectural Design*, 9.

the typical flat manner of the Egyptians. Alongside the plan is a note in Hieratic stating that the breadth is fifteen cubits.[75]

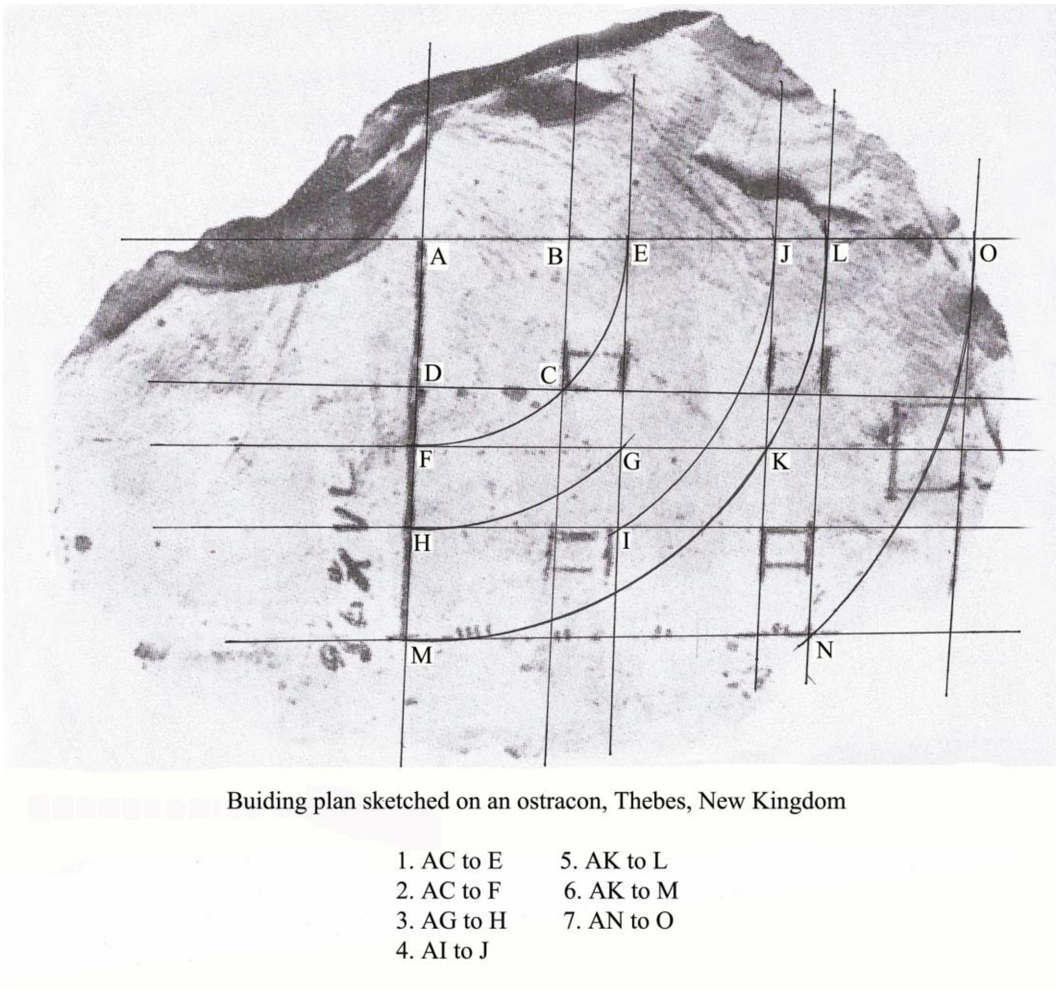

Fig. 42. Ostrakon from the Valley of the Kings, Thebes, on which a floor plan has been drawn.

The authors indicate there is also the numeral eight next to the long side, suggesting it is a fragment of a second note saying the length is 18 cubits. This cannot be the case as the perimeter of the building is a root rectangle and if the breadth is 15, the length is 15 times 1.412, expressing the relationship of the diagonal of the fifteen foot square to the side and therefore the length must be 21.18 cubits. That the perimeter of the

[75] Somers Clarke and R. Engelbach, *Ancient Egyptian Construction and Architecture*, Oxford: (Oxford university Press, 1930). Republished by Dover Publications, 1990.

building is a root rectangle suggests the possibility that plan-net analysis might be applicable to this relatively simple floor plan. The floor plan as seen in the photograph is not precisely rectilinear and the fit of the analysis to the plan in figure forty two is not perfect. However, it must be remembered that that the focal plane of the camera may not have been exactly parallel to that of the drawing and that an ostrakon is a broken piece of pottery and most certainly presented a slightly curved surface to the flat focal plane of the camera.

Allowing for these factors, the fit is surprisingly precise. Beginning in the upper left corner, an original square ABCD is marked out fitting to sides at AD and AB. Lines BC and DC determine a corner of the upper nearby column. Diagonal AC is rotated up to E and down to F, marking the width of the columns at C and I. The line FG marks a barely visible and somewhat irregular center line at F. Diagonal AG is rotated down to H to place a line marking the upper edge of the lower two columns. Diagonal AI rotated up to J places a transverse line JK that determines corners with lines DC and HI that mark the position of the remaining two columns. Diagonal AK rotated up to L places a line that marks the width of the latter two columns and rotated down to M marks the full width of the building. In the final step diagonal AN rotated up to O places a line that determines the length of the building.

This is a root rectangle plan because diagonal AK rotated to L and M results in a square ALNM. The diagonal of this larger square rotated to O marks a root rectangle. The short side of the rectangle is the breadth of the building and the long side is the length of the building. We have not demonstrated that plan-net geometry was used to draw the plan on this ostrakon, but the above analysis does show that the plan could be produced by such a method.

There is other evidence that construction of monumental buildings proceeded from a plan that put on a flat drawing surface, whether it was a clay tablet, a papyrus or a scraped parchment sheet. Figure forty three shows a statue of Gudea, the Babylonian king who ruled over the city of Lagash and its territory from 2144 to 2124 BC. He is seated on a stool and in his lap is a clay tablet on which is inscribed an architectural plan with a stylus and a graduated ruler. The tablet probably shows the plan for the walls around his temple Enninu. The graduated rule is damaged but it shows sixteen sections each of which is graduated from one to six, separated by an empty space. The Babylonian number system was in base six. That the plan is accompanied by a graduated rule is a practice that was continued right into the Early Modern period. Like the Early Modern examples, the drawn floor plan lacks detailed measurement expressed by numbers. The graduated rule served as the means by which the size of individual elements of the plan could be determined using a divider. If the building process was not governed in actual construction by cord and peg geometry but by calculation instead, then the procedure would be to copy a

Fig. 43. Gudea, King of Lagesh, ca. 2200 BC, seated with a building plan. Photo courtesy of Lessing Photo Archive.

distance off the drawn plan and check its length in terms of the standardized units on the graduated rule. The Babylonian mathematical system had the means to do such calculation, but factors such as the lack of a place-keeping decimal point and the indeterminate nature of Babylonian mathematical drawings associated with problem solving argue that it was unlikely.[76]

There is some evidence in cuneiform texts of a Babylonian affinity for building plans and cord and peg layout. Richard Ellis in *Foundation Deposits in Ancient Mesopotama* argues for foundation pegs as ritual objects rather than indicative of cord and peg geometry, saying

> It looks very much as if the original meaning of the peg was lost in the course of time, and that the representational part of the deposits was reinterpreted.[77]

This is most certainly the case, as the pegs in Ellis's study are found buried beneath some portion of the foundation, they often have decorative heads in the form of statuary and sometimes are ceramic. However, the fact that such pegs are ritual objects and are a reinterpretation of earlier usage indicates a strong connection to the tradition of cord and peg geometry in laying out ground lines for buildings.

Ellis records a prayer that indicates that Babylonians thought in terms of a plan for a dwelling. A ritual for the dedication of a house contains the following.

> Šamaš, bless this house which so and so, son of so and so, has built! Decree a good fortune for it. Draw a good plan for it! May this be a brick which brings well-being to its maker; may this be a house which brings well-being to its maker![78]

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[76] For indeterminacy in Babylonian problem solving, see Eleanor Robson, "Words and Pictures: New Light on Plimpton 322", *The American Mathematical Monthly* 109, no. 2, (February, 2002), 104-120.
[77] Richard S. Ellis, *Foundation Deposits in Ancient Mesopotamia.* (New haven: Yale University Press, 1968), 77.
[78] Ellis, *Foundation Deposits,* 186.

Inconsistency between the past perfect verb tense of "has built" and the future action implied in the imperative of "draw a good plan for it" suggests belief that the future unfolded according to a plan drawn up by deity just as the construction of a house unfolds according to a plan drawn up by the builder.

At this point it is clear that the ancient Babylonian design processes could result in drawn floor plans with which a measured scale may be associated, that there was a ritual connection between the foundation of houses and pegs to be driven or buried in the ground under the foundation and that making a plan was associated with a new dwelling. . In ancient Egypt lines were staked out on the construction site by a process that is described in foundation texts as "casting the plan-net upon the ground." The *nbi* stake figured in this process at both ends of the cord. In the previous chapter the relevance and possible uses of the square and the related figures of the circle and triangle were presented, the idea of sequential proportionality was established in the Canon of Polykleitos and the methods of laying-out by cord and peg was described. Analysis of the floor plan on the Thebes ostrakon showed that these considerations are relevant if the ostrakon plan were laid out by the method described at the beginning of this chapter.

In the intervening centuries between the Ancient World and the Early Modern world there is some textual evidence for the use of such geometry in laying out ground lines. According to the *Vita sancti Oswald IV,* in the tenth century Oswald

> sought most keenly for masons (*cementarios)* who would know how to set out the foundations of the monastery in a proper way, with the straight line of the rule, the three-fold triangle and the compasses (*qui recta rectitudine regulae et triangulo ternario alque circino scirent honorifice monasterii fundaments exordiri*).[79]

[79] John Harvey, *The Medieval Architect,* (London: Wayland Publishers. 1972), 107.

The text here suggests that it is straightness of line that is associated with the rule rather than measurement by modular units. Three-fold triangles, that is, 3 – 4 – 5, suggest either solid metal or wooden triangles to set angles or the use of a cord marked at three, four and five units as noted in the work of the Sutra technicians. The reference to the compass supports the conclusion that some of the work was based on geometrical constructions. The iron compasses used for such a purpose could be quite large.

In the twelfth century Giraldus Cambrensis, Gerald the Welshman, in *De rebus a se gestis* described his dream of Henry II's son John laying out a new church in Ireland, where he is described as making lines over the site to draw out the plan of the building

> . . . in a green plain. . . after the fashion of surveyors . . . marking the turf, making lines on all sides over the surface of the earth, clearly drawing the plan of a building.[80]

The *Methodus Geometrica*, published in Nürnberg in 1598 by Paul Pfintzing contains some 45 pages of text and copious illustrations on the right way to use geometry in land survey and measurement.[81]

There are two iconographic sources supporting the use of long cords for laying-out tasks such as were seen in the Sulva Sutra examples. The first is one of two miniatures in a twelfth century chronicle of the life of Abbot Hugh of Cluny. It illustrates the role of the monk Gunzo in the building of a new abbey church, Cluny III. Carolyn M. Carty discussed this image in "the Role of medieval Dream Images in Authenticating

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[80] Gerald. *De Rebus a se Gestis.* In H. Butler, *The Autobiography of Giraldus Cambrensis*,(London, publisher not given, 1937).

[81] See Elisabeth Pfeiffer, *Der Kartograph Paul Pfinzing Als Metrologue in Seine Methodus Geometrica und Bei Seinen Karten.* (1983).

Ecclesiastical Construction".[82] In figure forty four Gunzo is paralyzed and will be healed if he memorizes the plans for the new church that are about to be revealed to him by Saints Peter, Paul and Stephen and if he then relates them to Abbot Hugh. Carty notes that above him "the three saints roll out the measuring ropes against a gold background in order to reveal the ground plans that Gunzo is to memorize and convey to Abbot Hugh".

Fig. 44. Gunzo dreams a plan for Cluny in which Ss. Pter, Paul and Stephen lay out a network of cords.

Konrad Hecht writing in *Mass und Zahl in der gottischen Baukunst* says that the ground plan of this church was laid out with cords and that the layout occurred in two work steps. First the initial step is presumed to be with the help of a proportional figure, that is, with cord and pegs determining the points of the ground plan. In the second step he says the straight lines of the foundation trench go out from these points marked with cords, the line cords taking their course in relation to one another like the straight lines of the foundation, respectively parallel and at right angles to each other. In the miniature one sees the cord network spanning above the saints with lines parallel and at right angles to each other.

[82] Carolyn M. Carty, "The Role of Medieval Dream Images in Authenticating Ecclesiastical Construction.," *Zeitschtift für Kunstgeschichte,* 62, Bd. H. 1. (1999) 80-83.

Up to this point Hecht described the process to lay out a network of rectilinear lines on the ground with cords. He notes correctly that the intersection nodes of the network constitute points from which the lines of the plan go forth. His description is consistent with the "casting of a plan net upon the ground." He goes on to say, however,

> That the first step is worked out with cords, that is, that each measuring point is gained from a proportional pattern, is not supported by the miniature. Or, asked from the other side of the question, "If the miniature does in fact render a proportional pattern, what would the proportional pattern be that exists as some parallel and some rectilinear lines?[83]

He argues here that the narrative implied in the illustration "does not allow any motive to seek in the miniature the representation of a proportional method" and furthermore "that the length and the breadth (*longitudinis atque latitudinus quantitas*) of the church are measured (*metiri*)". He implied here is that the term 'measured' can only mean quantifying an interval by counting and adding units on a graduated rod. His argument is that absence of reference to proportional method implies its non-existence. He then asks, "If the miniature does in fact render a proportional pattern, what would the proportional pattern be, that exists as some parallel and some rectilinear lines?" I argue that the proportional pattern that exists as some parallel lines and some rectilinear lines is explained as the developing plan-net. He concluded that the surveyor's rod was so grounded in use at the construction site that it was indispensable. Though he was correct that the surveyor's rod was well grounded on the construction site and for certain purposes was the preferred tool, it was not the indispensable tool for every kind of design and lay-out activity.

[83] Konrad Hecht, *Mass und Zahl in der gotischen Baukunst*, (Hildesheim: Georg Olms Verlag, 1997). 220 – 221.

Nigel Hiscock covered much the same ground in his 2000 publication *The Wise Masterbuilder* and was sympathetic to the use of geometry. He focused on the 1 : √2 relationship that we saw is the proportion of the root rectangle, noting that it appears in Vitruvius, the sketchbook of Villard de Honnecourt, the late medieval handbooks of German master masons and in tables of numerical approximations to √2.[84] He cites 'a great weight of building analysis which shows close approximation to various square root of two permutations in Romanesque and Gothic architecture', all of which resulted in the 1 : √2 rectangle being hailed as 'the true measure' that was preferred in Medieval design.

Arguing contrary to Hecht's allegiance to an *ad quadratum* design scheme based on a grid of squares, he noted the common occurrence of layouts that involve incommensurable numbers and demonstrate variation in the spacing of arcade piers. He wonders at the preference for such systems that render expression in arrayed numbers so very difficult.

> The exercise of this preference therefore seems to point to the existence of a means of making the conversion (from incommensurable to commensurable and vice versa). Although the documentary evidence shows that the designing of a plan on parchment and laying it out on site both depended on the employment of geometry, the precise connection between design and dimension remains elusive.[85]

The precise connection between design and dimension that Hiscock refers to here reduces to the problem of change in scale from drawing size to ground plan size, something that when done by numerical calculation is lengthy, complicated and requires considerable calculating skill. When done by cord and peg geometry it reduces to an analogue process

[84] These approximations to √2 for various numbers are those approximations seen in the Babylonian array for determining the diagonal length for squares of the arranged numbers on page 21.
[85] Hiscock, *The Wise Masterbuilder,* 278.

that matches geometrical shapes in which there is no change in method from design steps to steps that lay out the ground lines. The only element that changes is the scale of the work. Divider and straight edge steps taken at the scale of the design drawing, whether on tablet, parchment or tracing house floor, are exactly the same steps as design by cord and pegs at construction scale. Though size changes, the aim is to match shape perfectly to shape. Keep in mind here Harvey's remark quoted earlier that the architect knew the traditional rules of proportion and geometrical methods of setting out a building in all its parts and details, from a single basic unit. The problem of scaling up from design to construction size reduces then to that of determining the right size of the initial figure to produce building ground lines of the right size for actual construction.

It is here that the measuring rod possibly came into play. To lay out altars with cord and pegs in Vedic India, Ṡulvasūtra technicians made numerically measured marks on the cord preliminary to laying out the altar. These marks along the cord were important in the geometrical process of laying out the altar.[86] For our purposes an initial figure at small scale can be constructed by measurement with a graduated rule to serve as the preliminary figure from which the whole plan will be extracted geometrically. There is no reason to think that the measuring rod was not used to create the initial figure whose scale was conveniently small, as well as later in the process for determining details such as window and door dimensions.

In answer then to Hecht and Hiscock, Hecht asked the question, "If the Gunzo miniature does in fact render a proportional pattern, what would the pattern be that is both

[86] For an understanding of the methods of the Sulvasutra technicians of Vedic India, see G. Thibaut. *Mathematic in the Making in Ancient India,* ed. with introduction by Debiprasad Chattopadhyaya, (Calcutta: K.P. Bagchi and Company, 1984). For the text of the Sulvasutra-s, see D.P Kularia, *Kātyāna Śulbasūtra with Commentary,* (New Delhi: Devesh Publications, 2009).

parallel and rectilinear?" The answer to this question is; The process of casting the plan net as proposed here does so by swinging the diagonal of an initial square figure to create a root rectangle formed of lines that are parallel and rectilinear, and then sequentially expanding this network of lines by repeating the process until all significant features of the plan are positioned and given dimension by either lines or intersection nodes of the plan-net. Hiscock asked what the relationship of dimensions might be between design stage and construction stage. Plan-net analysis demonstrates a method that conserves the identity of steps taken at the design stage with the steps taken at the ground line stage.

It is clear that a given house type was spatially organized by a specific set of steps followed in a specific pattern. The immigrant German designer/builder on the Pennsylvania frontier in 1745, planning to build his home as a thirty foot log three-room central fireplace house needed to remember that an initial square of 9 feet allows for a fireplace with a hearth of six to seven feet depending on how the jambs are arranged inside or outside the lines of the square. From this square a ground line plan of some thirty foot length could be generated, depending on the precise pattern of subsequent steps he chose to take. The answer to Hecht's question about the form taken by the process is demonstrated as the builder lays out on the ground a remembered pattern of six rotational steps that produces a network of lines marking out the three room plan. As will be shown for the Bertolet house in the next chapter, from a nine foot square there is extracted a network that forms a rectangle the sides of which are in the proportion of three to five, a width of 17.98 feet and a length of 29.82 feet. The width deviates from 18 feet by 1/50th of a foot or one quarter of an inch, and the length deviates across 30 feet by eighteen hundredths of a foot or by 2.16 inches. The lines of the network mark the position of the fireplace, the kitchen, the parlor and the multi-purpose room. When building in stone, one can simply lay stone “to the end of the line”. When building in log the lines staked out on the ground are easily copied with cord that serves as a template for marking logs to be cut to the correct lengths. Such a method would serve well in an environment in which tradition passed on a limited number of house types with a limited degree of variation from house to house to meet individual needs and desires.


Fig. 45. Norbert’s Vision of the Crucifixion.

The transition from practical method to symbolic remembrance was concurrent with the rise of literacy and calculating skill as these tended to replace apprenticeship with published texts as the source of learning. Some iconographic sources make this shift more explicit. The first two are a pair of copper engravings referred to by Konrad Hecht and dated ca. 1525 from the *Traditionskodex des Weissenau bei Konstanz.* Taken together they say much

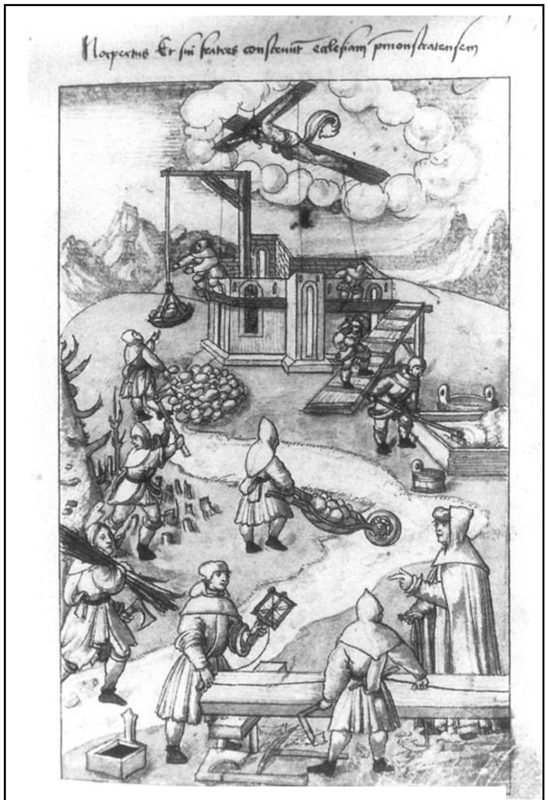


Fig.46. The Weissenau monks build their hilltop chapel.

about perception at that time of the function of cord and peg methods in transmitting *recht Mass,* the "right measure" in construction. Figure forty five shows Norbert's vision of the Crucifixion on the hilltop upon which the first chapel of the monastery is to be built. Rays of light transmit the vision's true narrative to the monks below. Figure forty six shows the monks building their chapel on that same hilltop and underneath the same vision in the clouds, from which cords, rather than rays of light, transmit the vision. The

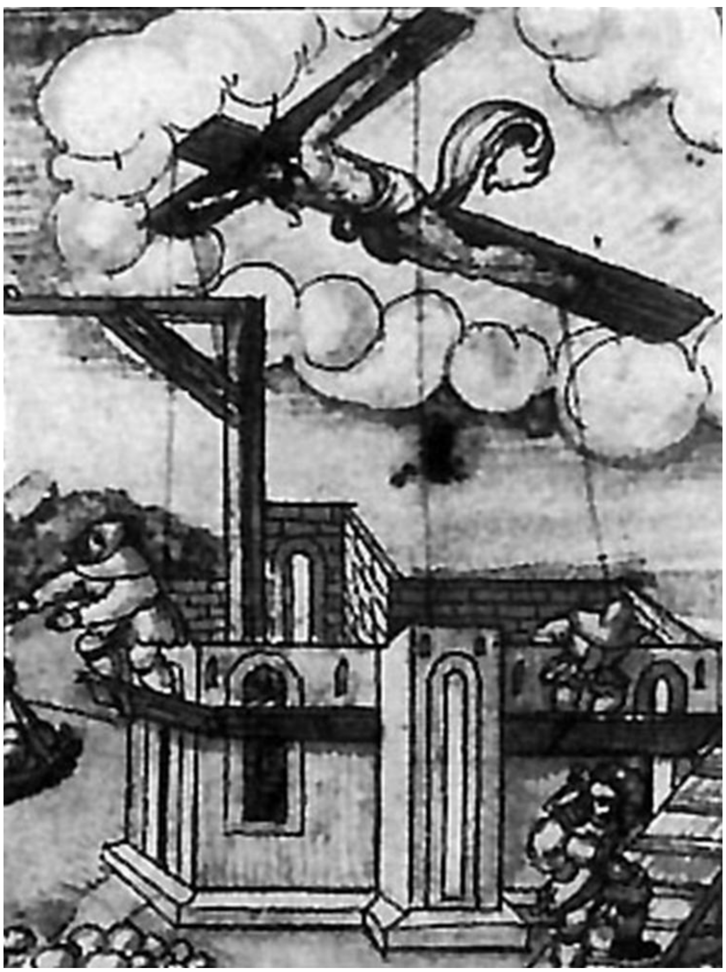


Fig. 47. Detail showing the cords connecting the vision in the clouds to the actual construction on the ground.

Master-builder stands in the foreground giving instruction to two workers marking a timber with red-chalked cord to cut it to size. The chapel rises in the middle ground of the illustration. Konrad Hecht correctly noted that the workers in the foreground are not triangulating a geometrical problem with the raised cord over the beam but he failed to comment on the visual narrative taking place at the top of the hill. Detailed in figure forty seven we see that there are four cords connecting the vision in the nimbus of clouds to the actual construction work on the ground. Just as the rays of light in Norbert's vision transmit nuanced meaning, these cords transmit connotative meaning that will be read by the contemporary viewer. Our task is to accurately grasp hold of that connotative meaning.

To understand how this illustration was read by contemporaries, we need also to be aware of some of the associations this visual image brought to mind for the contemporary viewer. In the words of Lori Garner comparing oral poetics and architecture, formulas of visual and oral composition "establish nuanced meanings through subtle manipulation of formulaic phraseology." In this way she says, visual and oral compositions become invested with connotative meanings. These characteristic formulas, though utterly familiar, are nevertheless explicitly stated visually and repeated frequently. They imply associative social values to which the viewer or the audience can respond.[87] The associations brought to this engraving by the contemporary viewer understood the Baumeister to be the channel through which a higher vision is transmitted. In this visual statement, the Baumeister transmits the vision in the form of instructions to the workers just as through the medium cords and pegs, the vision of "right measure" is transmitted to the Baumeister from the Divine Mind above. Contemporary viewers of this engraving would understand that it is transmits the vision in the form of instructions to the workers just as through the medium of cords and pegs, the vision of "right measure" is transmitted to the Baumeister from the Divine Mind above. Contemporary viewers of this engraving would understand that it is cords, physical or mentally imagined, that line out the plan, whether on paper, on the ground or in the mind of the designer. They transmit the "right measure" to the earth as ground lines and through the

[87] For a detailed examination of the parallels of Anglo-Saxon architecture and oral poetics, see Lori Ann Garner, *Structuring Space.* Notre Dame: University of Notre Dame, 2011.

associative skills of the audience, transmit the plan from the hidden structure of the cosmos to the inner vision of the Baumeister. The connective cords and their pegs in conjunction with the master builder in his instructive role and the corona on which the plan floats above the work, signal a complex of associative meanings that links the vision of Norbert to the construction of the chapel through the mind of the Baumeister in a manner that was within the associative capabilities of the illustration's contemporary audience.

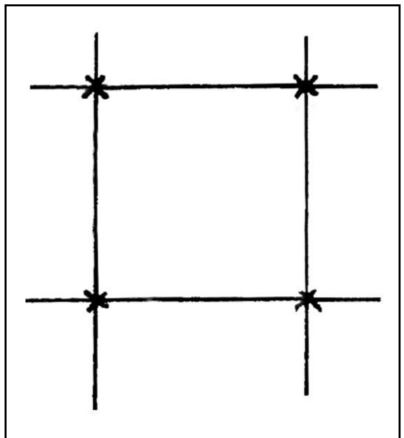


Fig. 48. The initial plan-net square

The role of cord and peg geometry that made possible a plan-net methodology was transformed from a practical role in architectural problem solving to the role of symbolic remembrance in the ritual practice of the Order of Freemasonry. This transition was a consequence of the broad rise of literacy and calculating skills that enabled use of arithmetic rather than geometric practices of design and control of construction processes. In figure forty eight we return to the initial square from which the plan-net expansion was demonstrated in figure thirty nine.

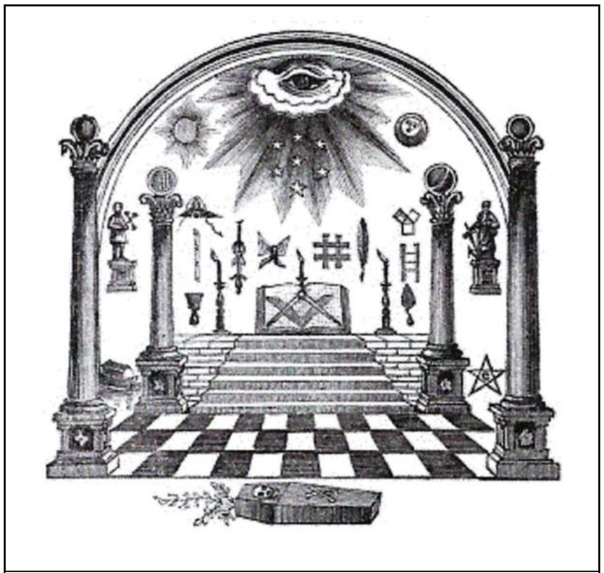


Fig. 49. The square as two pairs of parallel lines in a constellation of Freemasonry symbols of a degree on a Masonic apron.

Keeping this in mind we look next to a constellation of symbols on the apron of the Freemasonry movement in figure forty nine that preserves in remembrance tools and concepts vital to this method of plan design throughout its very long history. The set of images on the apron appears with in variety of different individual symbols but generally in the same construct with the altar at the center, the eternal eye above and pillars to each side. Above the altar on the right side is a familiar square defined by two pairs of parallel lines. Between the columns at lower right is the five pointed star seen in figure twenty six from the sketchbook of

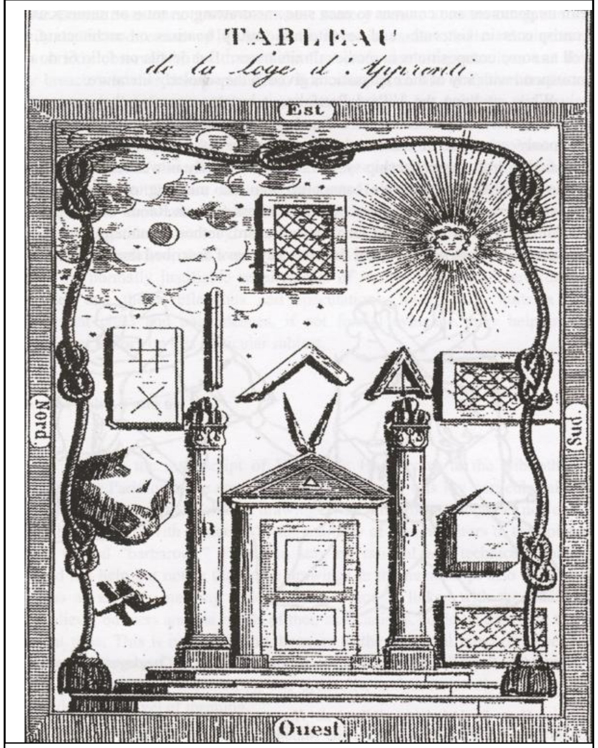


Fig. 50. A similar set of Masonic symbols from an eighteenth century French Masonic handbook.

Villard de Honnecourt. Left and right of the omniscient eye the circle so prominent in the reconstructed geometry of Tons Brunés is seen in the figures of the moon and the sun. Center at the far left and repeated at the top of the steps is the crossed figure that defines Brunés' circle. These are essentially the same kind of symbols as those of the first degree of Masonry on a page from an early nineteenth century French book on Freemasonry in figure fifty. Here the whole is wrapped in the knotted cord that served the Pharaoh and the priest of the goddess Seshat in the act of "casting the plan-net on the ground." Rather than the altar, at the center is the doorway to enter to the First Degree. Centered above the door are the divider and square, and over the columns are the straightedge and the leveling device. Framed top center and also to the right of the leveling device is the square and the rectangle framing a network of cords such as we saw previously in the illustration of Gunzo’s dream. On the tablet to the left of the straightedge are the two symbols mentioned just previously, the crossed lines defining the circle and the two pairs of intersecting parallel lines that define a square.

We have come full circle in this chapter, beginning with two pairs of intersecting parallel lines that define a square and ending with the same. Implied in this figure is a networkof rectilinear lines that can be expanded using a divider and straightedge or cord and pegs. Ancient Egyptian foundation inscriptions contain texts that indicate use of a net of lines for casting building plans on the ground as ground lines. The twelfth century ‘Dream of Gunzo’ illumination depicts just such a network and the fifteenth century *Traditionskodex des Weissenau bei Konstanz* provides a broad understanding of the role of cord and peg methods in transmitting the designer/builder’s vision as a plan upon the ground. By the eighteenth century the method and its tools become formalized as symbols bearing the remembrance of past practices. The square, the network of lines that can be extracted from it and the application of this network to the analysis of vernacular floor plans merit our full attention in the next chapter.

CHAPTER V

The application of Plan-net Analysis

In this chapter plan-net analysis applies an easily remembered and executed geometrical design method to lay out the floor plan of a building. Techniques of this kind survived from the Ancient World into the Early Modern because they were simple to carry out and reliable. We should keep in mind some considerations established in earlier chapters. All of the following analyses begin with an initial square base figure small enough that if preferred, it could also be measured out with a modular measuring rod and its corners made true with a square. The completed floor plan is extracted geometrically from this base figure, every element derived from a previously constructed element. Architectural details not marked by newly generated plan-net lines or their intersection points are marked by copying a plan-net interval from one location to an adjacent location, or by dividing an existing plan-net interval by halves or by fourths. Small detail elements not laid by plan-net methods are measured out by rule from previously placed plan-net points. Floor plan elements may fall inside and/or outside of lines but are not centered on a line. In most plans, the initial base figure is a square that marks some or all of the hearth dimensions.

Fig. 51. The ca. 1745 Bertolet house, Berks County, Pennsylvania.

The Bertolet house in Berks county, Pennsylvania house was built by a recently immigrated settler, probably in the decade of the 1740s, on a floor plan widely found on the undeveloped and forested landscape of early Southeastern Pennsylvania. How one gets from a small initial square base figure to a completed building floor plan can be demonstrated using a plan drawn by Henry Glassie for his article in a 1968-69 issue of *Pennsylvania Folklife*.[88] This plan, illustrated in figure fifty two, is that of the three-room Germanic central fireplace house as described in detail by Richard Weiss in *Hauser und Landschaften der Schweiz.*[89] The three rooms

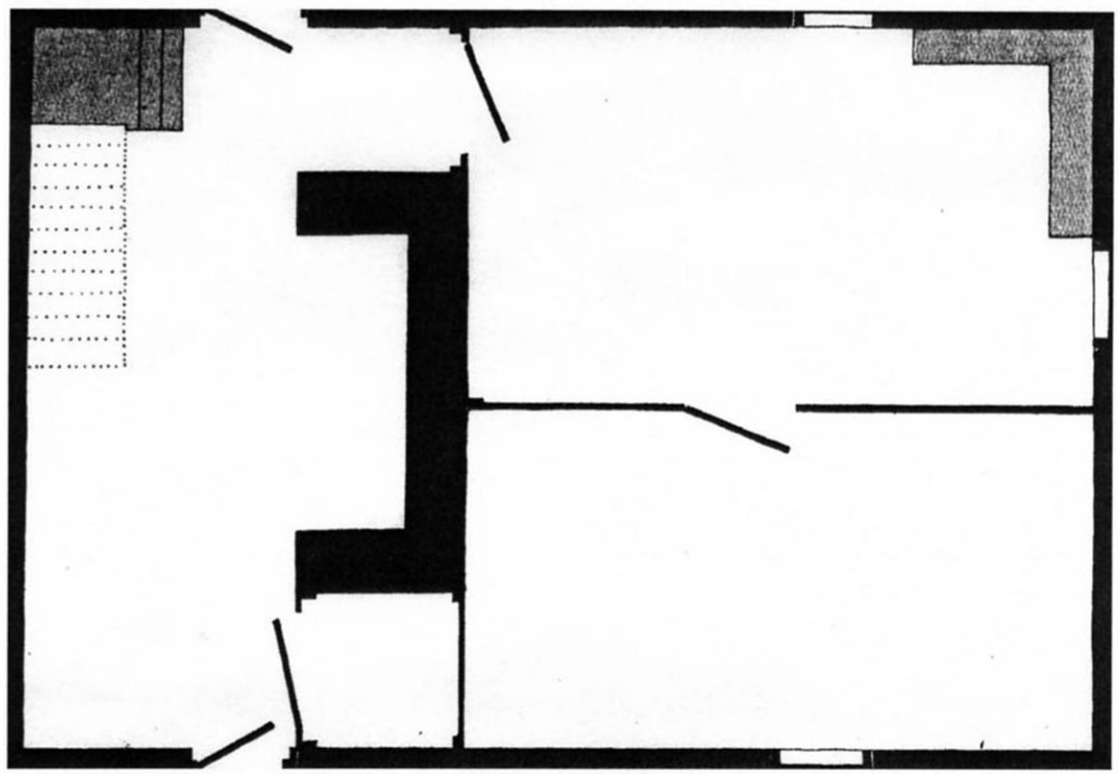


Fig. 52. Floor plan, Bertolet House, Berks County, Pennsylvania. Drawn by Henry Glassie.

wrap around the asymmetrically placed hearth. The asymmetrical facade and plan suggest the possibility of geometric organization rather than organization by modular measurement. The horizontal *Stube/Kammer* partition centers neither on the fireplace back wall nor on the right gable of the house, a point of some significance in concluding the following analysis.

[88] Henry Glassie, "A Central Chimney Continental Log House," *Pennsylvania Folklife,* Lancaster: The Pennsylvania Folklife Society XVIII, no. 2, (Winter, 1968-1969). 32-39.
[89] Richard Weiss, *Hauser und Landschaften der Schweiz,* (Erlenbach-Zurich: Eugen Rentsch Verlag, 1959). 132-155.

The initial square base figure almost always relates to some aspect of hearth placement and/or size, so to begin the analysis, a search for the base figure begins at the hearth, illustrated in figure fifty three. A square can be constructed that incorporates lines of the plan on all four sides, the fireplace back wall line BC, one jamb on side DC inside

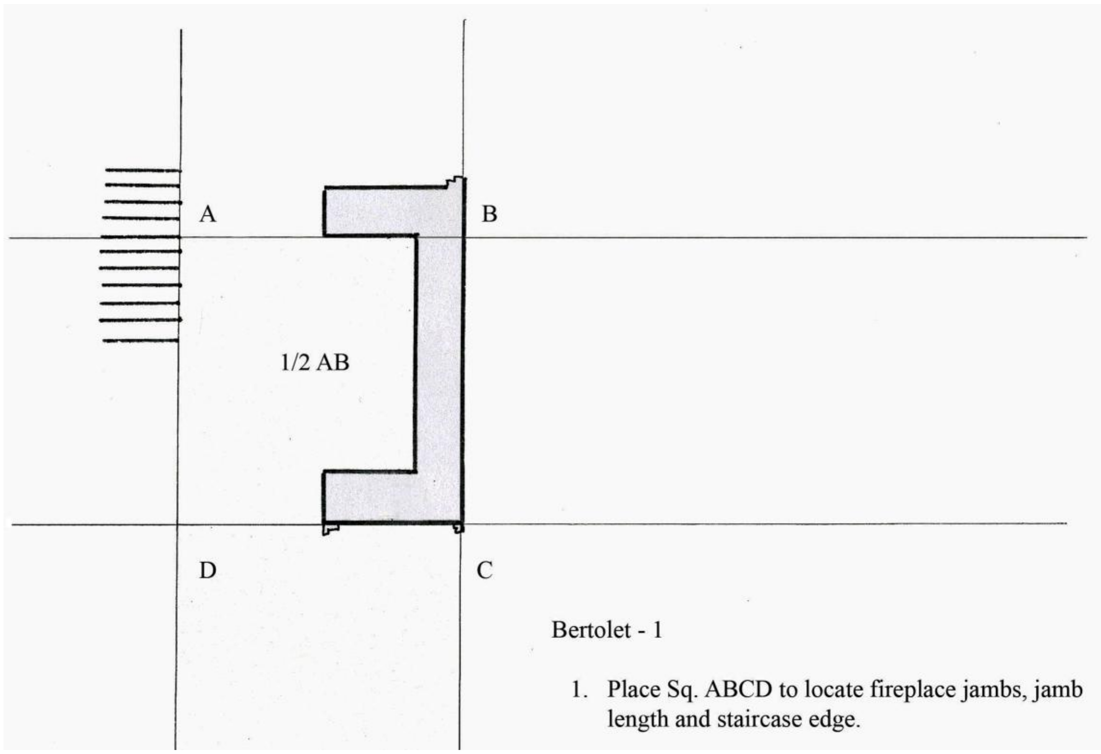


Fig. 53. The initial square base figure for the extraction of the Bertolet floor plan.

the square and on the other side at AB outside the square. AD marks the edge of the staircase. The initial square base figure is now identified.

Diagonals AC and BD are rotated in all directions to see if they mark additional features of the plan. In Figure fifty four, rotating diagonal BD down to E marks the location of the front wall. BD rotated up to F marks the location of the left gable wall. Use of the divider demonstrates that that the front door is located at ½ of side DC of the base figure, the same point at which the two jambs terminate. Line DA and the left gable line at F determine the width of the staircase. Every feature of the plan determined thus far is governed by the lines of the plan-net analysis.

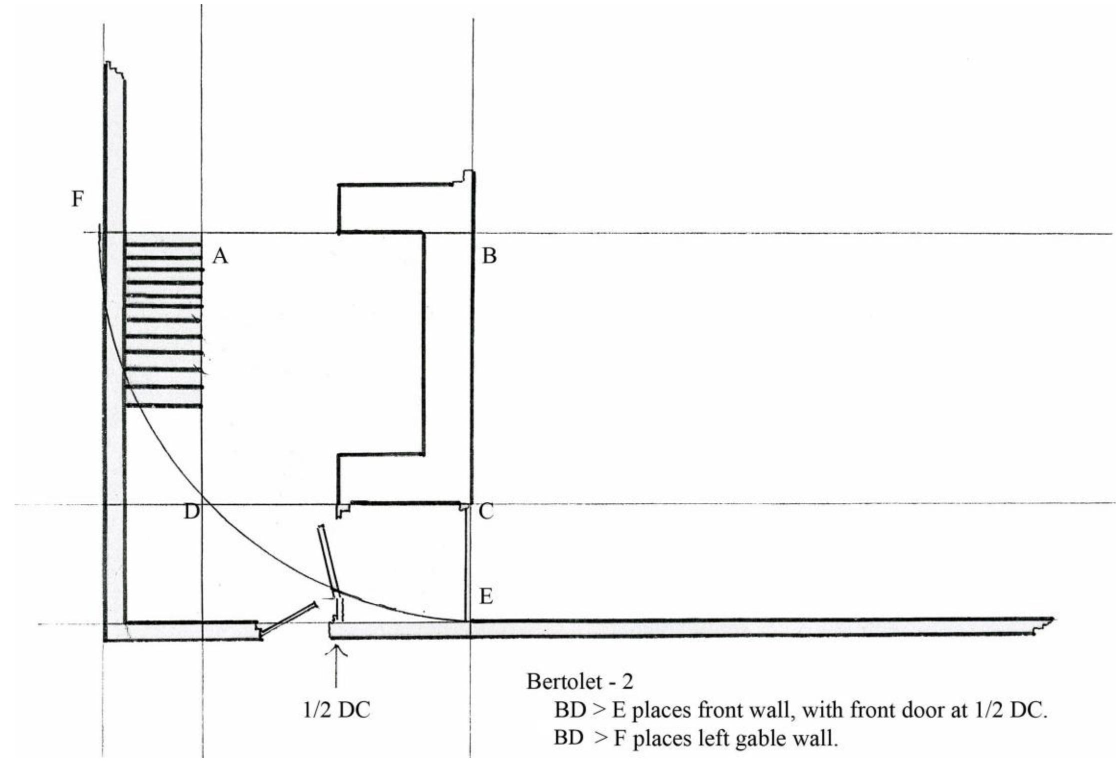


Fig. 54. Diagonal BD rotated to E marks the front wall line and rotated to F marks the left gable wall line.

In figure fifty five diagonal GB rotated up to H mark the turn of the staircase at the landing and GI rotated up to J marks the location of the rear wall, with the rear door at ½ of base figure side AB, the line also marked by the front door and the two jamb ends. These two steps have established the depth of the house on the transverse axis. This starting pattern is common in houses of this type. The base figure diagonal is rotated in two directions establishing the first corner, and then the plan is extended transversely by rotating the diagonal of successive rectangles to mark the back wall, establishing the width of the building.

Throughout the process of extracting the plan-net there develops an increasing cascade of nodes available for the next move. The base figure has four

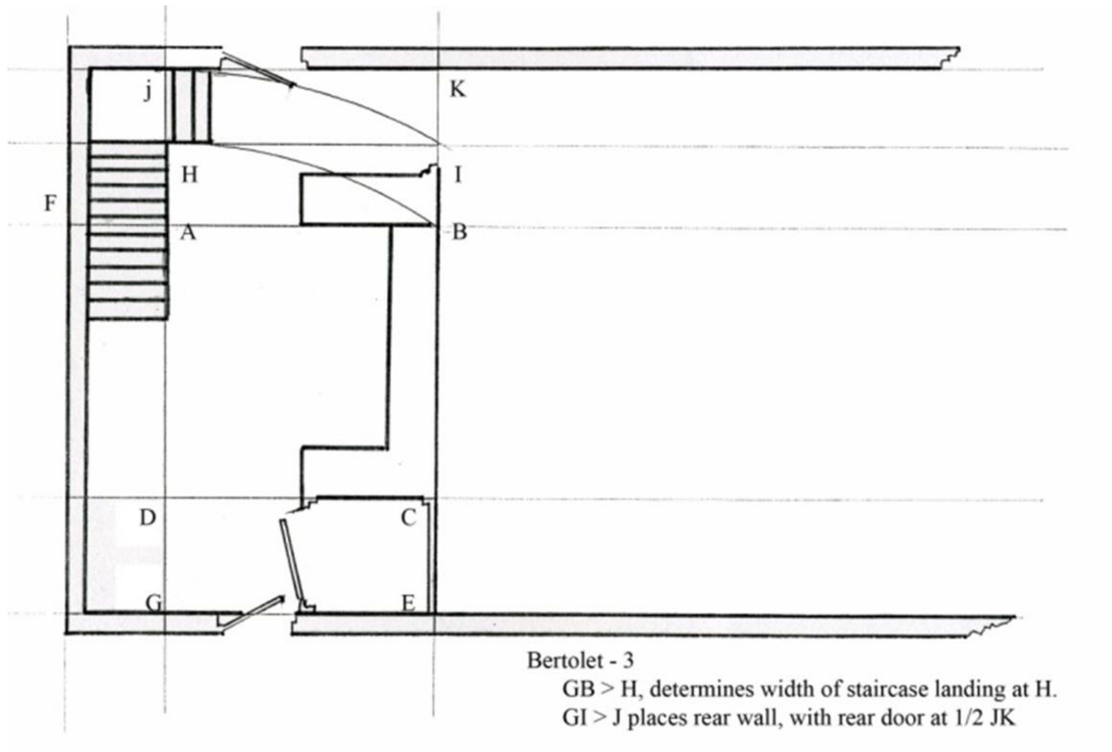


Fig. 55. Two expansion steps on the transverse axis serve to mark the rear wall line at J and the turn of the staircase at H.

intersection nodes. In figure fifty four there are nine nodes. By figure fifty five there are fifteen nodes. The completed plan-net in figure fifty six has twenty five nodes. Thus as the network develops it becomes increasingly possible to shape what is possible to what is desired.

The two transverse steps shown in figure fifty five are followed in figure fifty six by two lateral steps to complete the floor plan. Diagonal GK rotated to L marks the position of the front and rear windows. Diagonal MN rotated to O marks the right gable wall. The gable window is marked by the extension of line AB and the adjacent wall bench in the corner where the table is found is marked by the same line.

The only feature not marked by plan-net lines and nodes is the Stube/Kammer partition. Careful measurement with a divider shows that it is neither at the center point

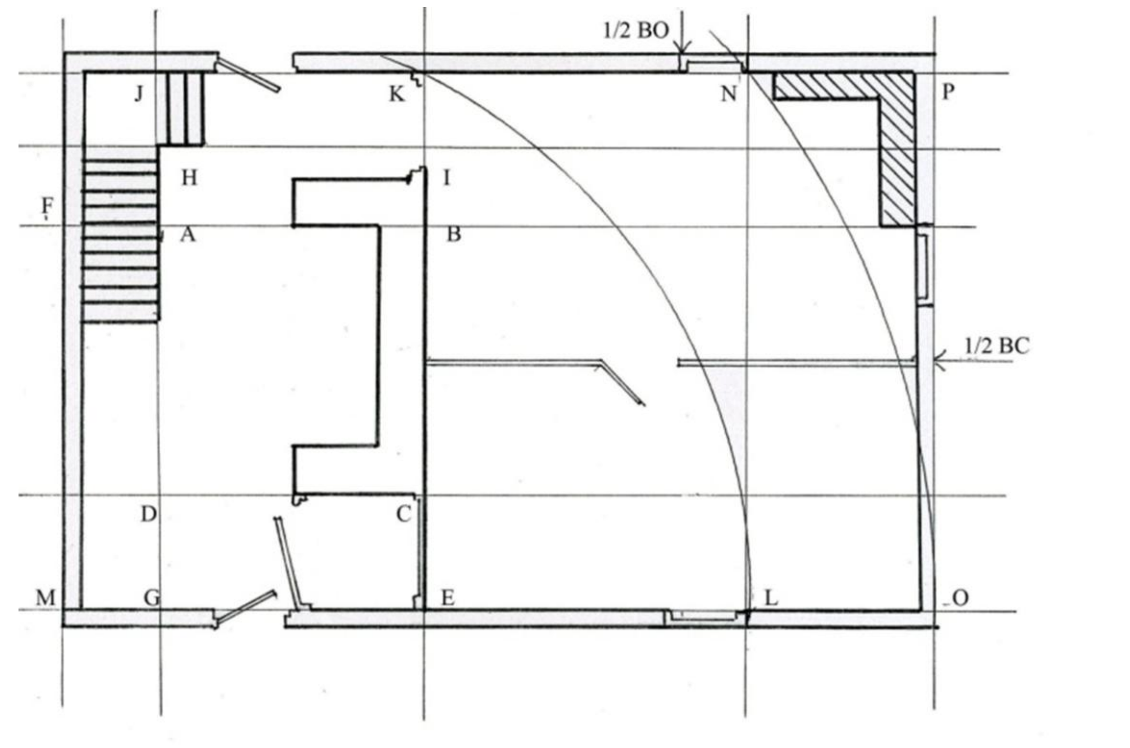


Fig. 56. The completed plan net for the Bertolet house, marking the location of all significant elements of the floor plan.

of the right gable wall nor does it center on the rear wall of the fireplace. Rather it marks the center point of side BC of the square base figure. This could be coincidence, but given the precision with which all significant architectural elements of the plan are marked by the lines and nodes of the successively generated plan-net it is more likely that the square base figure was in fact the beginning point of plan development and the partition centered on it.

The Bertolet plan development demonstrates a number of characteristics common to this house type; 1. The initial square relates to the location and size of the fireplace. 2. Plan development begins by fully developing the kitchen area. 3. This determines the depth of the house by marking the limits of the transverse axis. 4. Plan development then moves to the lateral axis to determine the length of the house. The plan is unified throughout by the root rectangle relationship of diagonal to side that is common to every stage of the plan development.

The plan development process matches an existing element to a new space, to determine a figure that is constructed in a new location. This process is analogue in nature, that is to say, it uses one physical shape as the model for a shape in another place. This can be done in two ways. The diagonal of an existing quadrilateral may be fit to a new location to generate a new and larger quadrilateral from a smaller one. Alternately, an interval already marked on the plan-net may be copied to an adjacent location by rotating that interval around one of its end points, either linearly on the same line or perpendicularly around a corner. Both approaches will be found in the analyses that follow, the former more frequently in laying out the general schema of the plan and the latter often used to determine divisions within the overall plan and to place windows and doors.

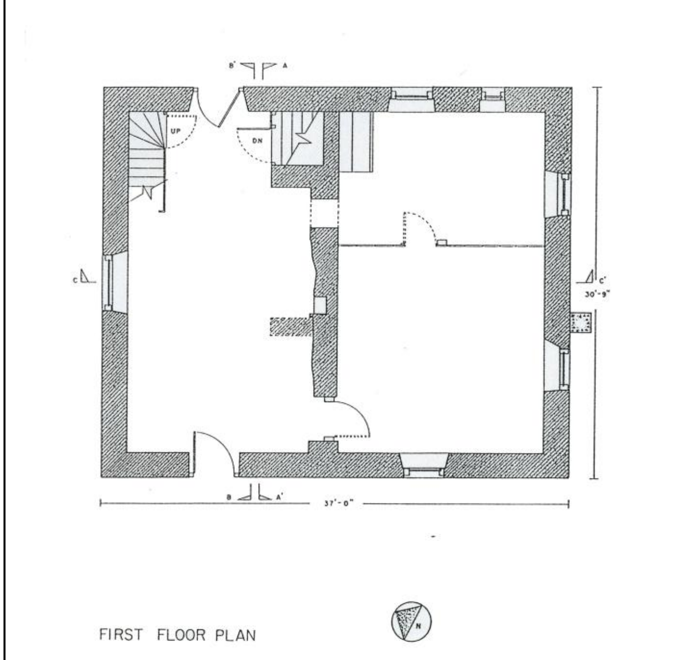


Fig. 57. First floor plan of the Henrich Antes house. Historic American Buildings Survey.

In 1736 Henrich Antes designed a two story stone house in Montgomery County, Pennsylvania having the same floor plan though proportioned a bit differently and more expressive of his *Kleinstadt Būrger* roots in *Freinsheim* in the *Rheinpfalz.* This floor plan was widely adopted by early eighteenth century immigrants into Southeastern Pennsylvania. Antes' version of it begins in the same manner but is modified to produce a somewhat different result. The kitchen area is transversely oriented along the left gable wall, there is a square parlor on the right front side and to the rear of this is a smaller room laterally oriented, all differently proportioned from the Bertolet house but essentially the same arrangement. Through the house from front to back is a massive stone wall rising to attic level locally called the *Feuerwand*, the fire wall.

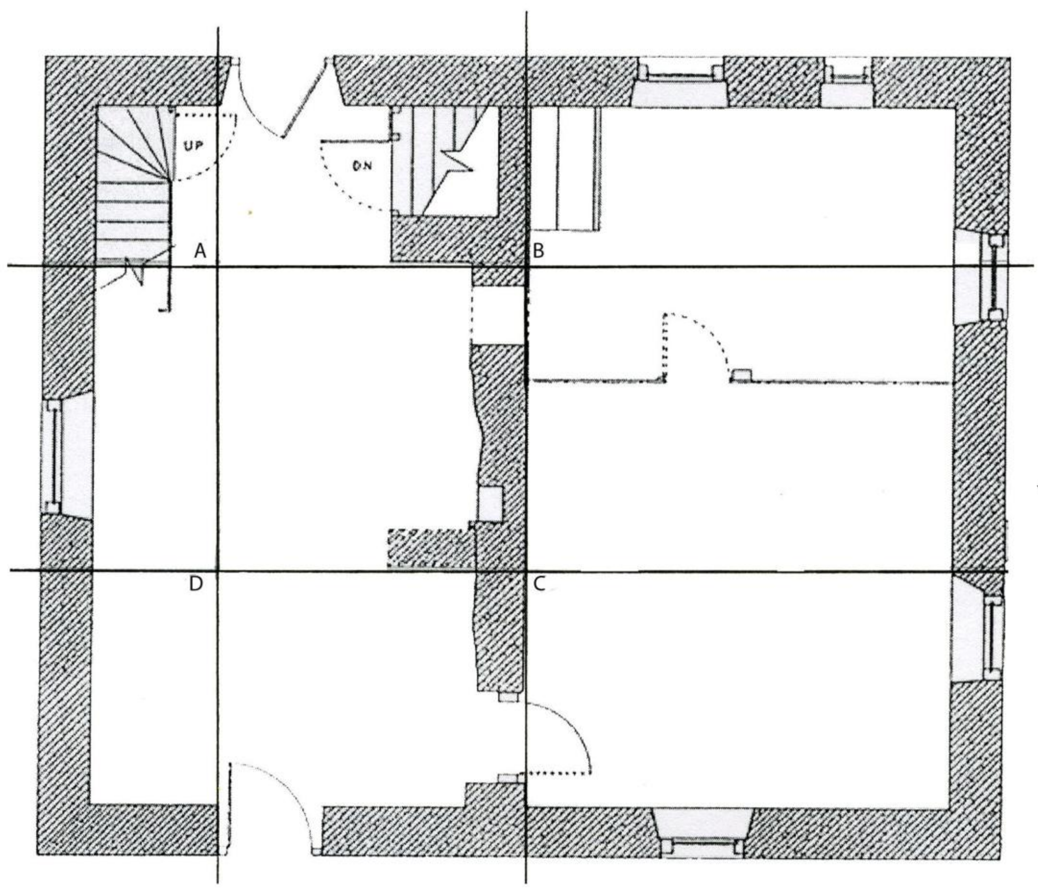


Fig. 58. Antes house, the initial square base figure in relation to the fireplace and other features

To begin the analysis, in figure fifty eight a square is sought based on the dimensions of the fireplace that will incorporate some other element of the plan. The AD line of the square aligns with the front and rear doors. The fire wall carries through from front to back along the BC side of the square. In figure fifty nine as in the Bertolet house, diagonal BD is rotated to E and to F though the result is slightly different. Where in the

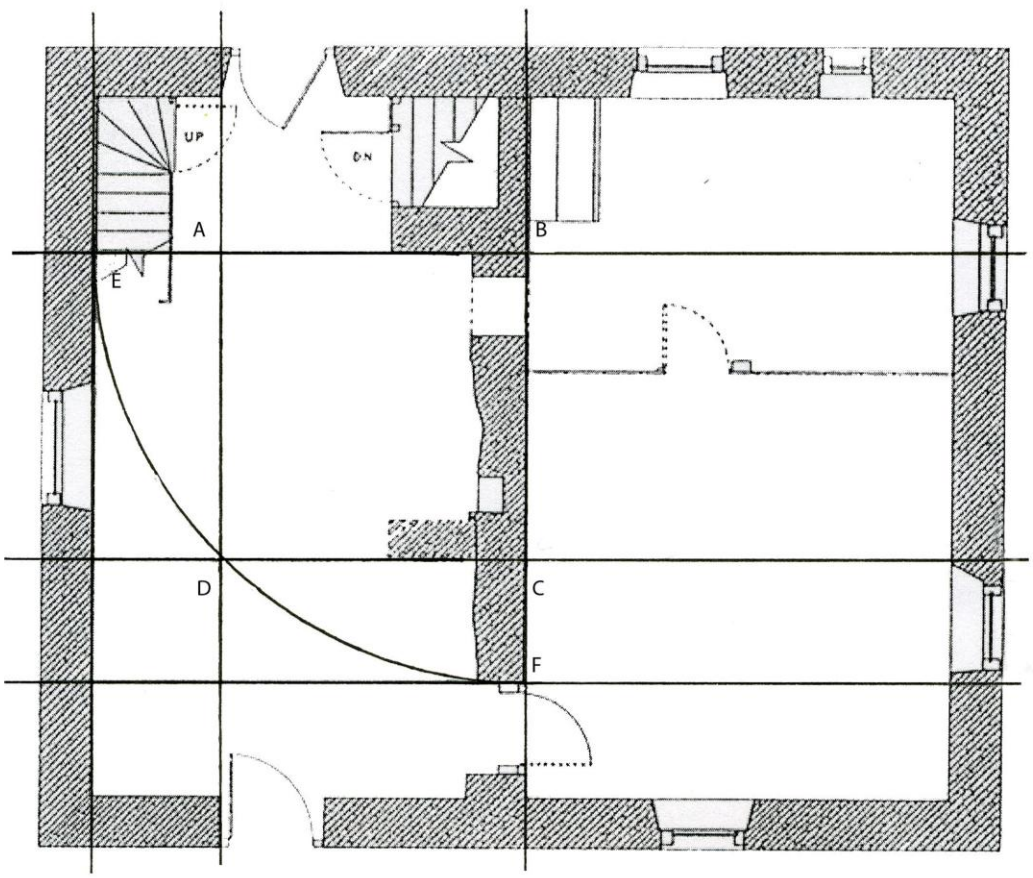


Fig. 59. The first two expansions duplicate those of the Bertolet house, but with slightly different results in the overall plan.

Bertolet house the rotation toward the front marked the front wall line, in the Antes case it marks the door through the firewall into the parlor. Because there is another rotation yet to come toward the front in the Antes house, this plan will be squarer than is that of the Bertolet house. In both cases however, the rotation to the left marks the left gable wall.

In figure sixty the next two steps complete the kitchen area of the plan just as it was the first part of the plan to be completed in the Bertolet house. Increasingly longer

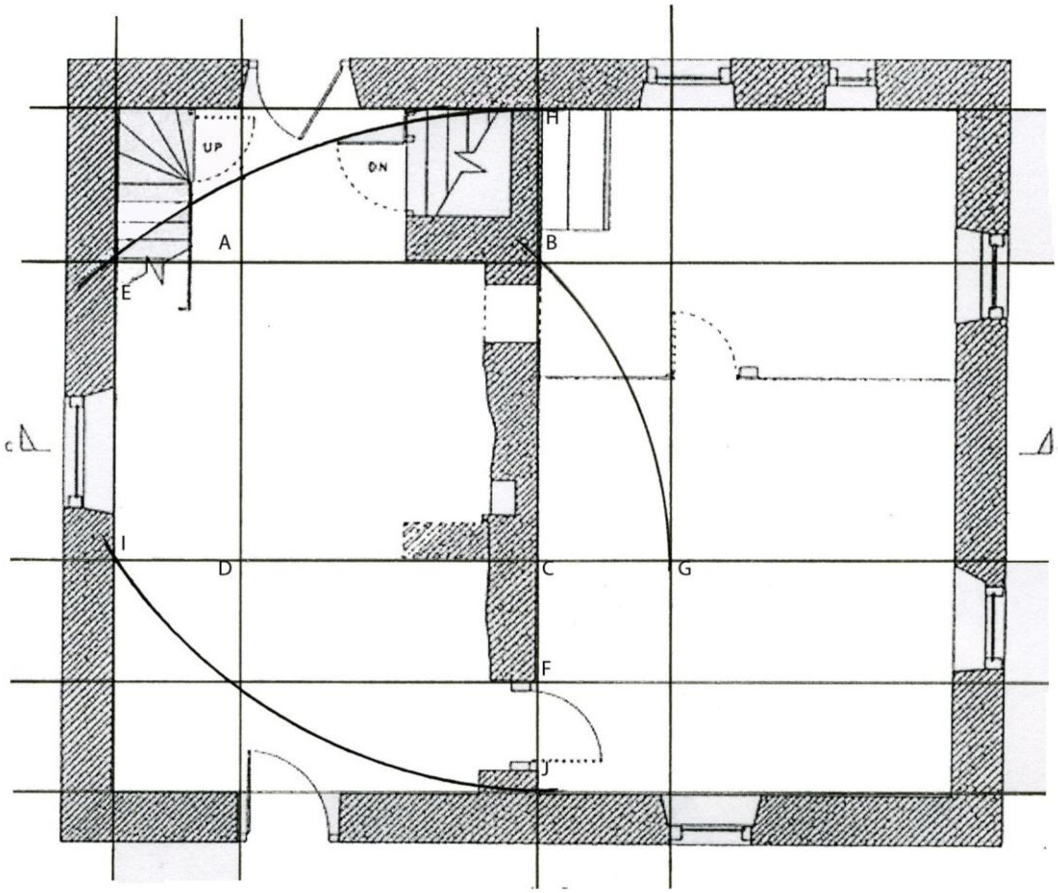


Fig. 60. Successively longer diagonals BI and then JE, serve to mark out a more spacious kitchen area.

diagonals are chosen. BI is rotated down to J to mark the front wall line, and then JE is rotated up to H to mark the rear wall. The longer intervals BI and JE serve to create a more spacious kitchen to meet the needs of Henry Antes' public life. Diagonal DB of square ABCD rotated down to G then marks a line for the front parlor window and the *Stube/Kammer* partition door.

It is at this point that Antes deviates from the pattern completing the plan as seen in the Bertolet house. Bertolet made two larger diagonal rotations to the right on the lateral axis to mark the right gable wall. In figure sixty one Henry Antes copies interval HK on the rear wall line over to HL to mark the right gable wall line thus doubling the size of the plan. The HJ line along the firewall serves as a center line in a pattern that is much squarer than that of the Bertolet house. The interval MJ is copied around the corner at M to mark the partition for a square parlor at O.

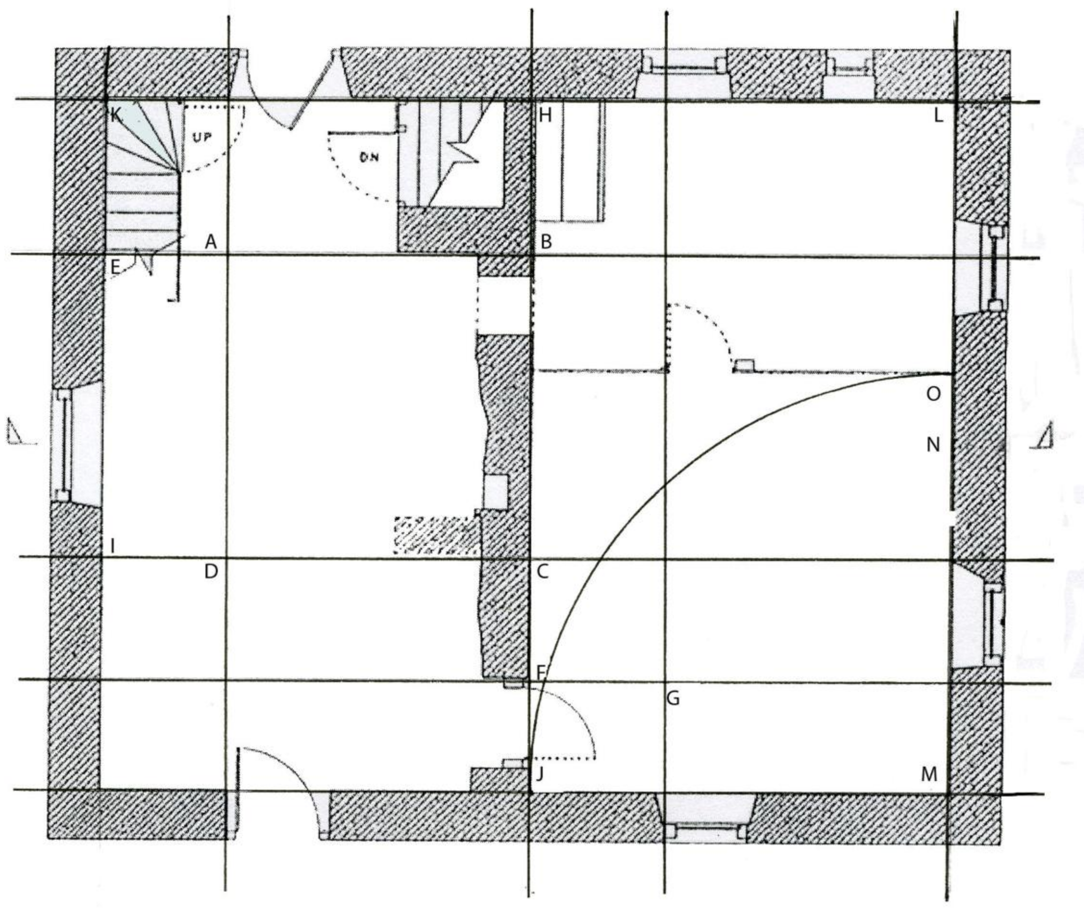


Fig. 61. KH is copied to HL to mark the right gable wall line. The interval MJ is copied around corner M to MO on the right gable wall line to mark a square parlor.

Small flags right and left mark the lateral centerline of the plan. The left gable window is centered on this line. On the right gable the front-most window is centered at one half of OM and the rearward window is centered at one half of LN. The small rear wall window to the right was demonstrated in the restoration of the building to be added later.

Though the Bertolet house and the Antes house show the same three room central fireplace floor plan, the former is a one and a half story frontier dwelling of log and the latter is a stone house of two full stories with double attic. Plan-net analysis shows essentially the same thought pattern in the plan development of both, but with slight variation in approach resulting in a differently proportioned pattern. Henry Antes is best understood as mediating Old World tradition in the New World environment as an agent of transition. Architecturally his house points backward and forward. With a traditional floor plan identical to the Bertolet house, Antes symmetrically balanced his asymmetrical central fireplace three room interior across this center line. Right and left gable windows are symmetrically placed but the facade shows the same asymmetrical pattern as does the Bertolet house. The Bertolet house is completely asymmetrical and traditional in all its aspects. The geometry of Henry Antes' house, looking backward with its traditionally rooted asymmetrical floor plan to a vernacular

Fig. 62. The asymmetrical Antes house facade, noting the door and chimney locations.

architectural style, communicates a transitional state when at the same time its symmetrical aspects communicate the rising social position he and his family achieved in a New World and that looking forward, anticipates the coming Pennsylvania Georgian style soon to sweep across the landscape.

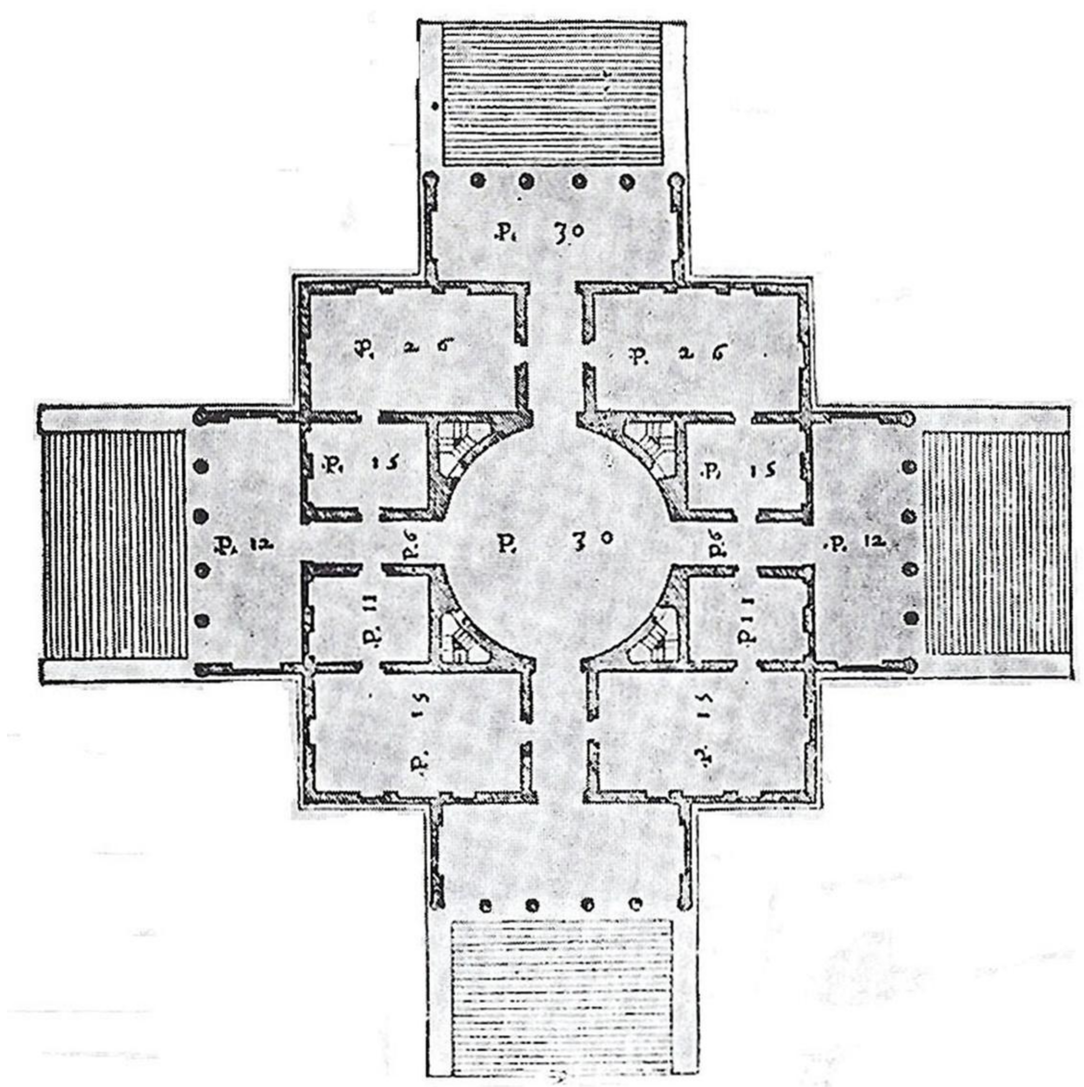


Fig. 63. Andrea Palladio’s plan for the Villa La Rotunda, 1570.

Similar four-directional expansion from the initial square results in an entirely different plan seen in an elegant design by Andrea Palladio, the 1570 Villa La Rotondo. This plan is symmetrical around its center point. Generation of the plan-net begins with the circle circumscribed by a square as was described by Tons Brunés in an earlier chapter.

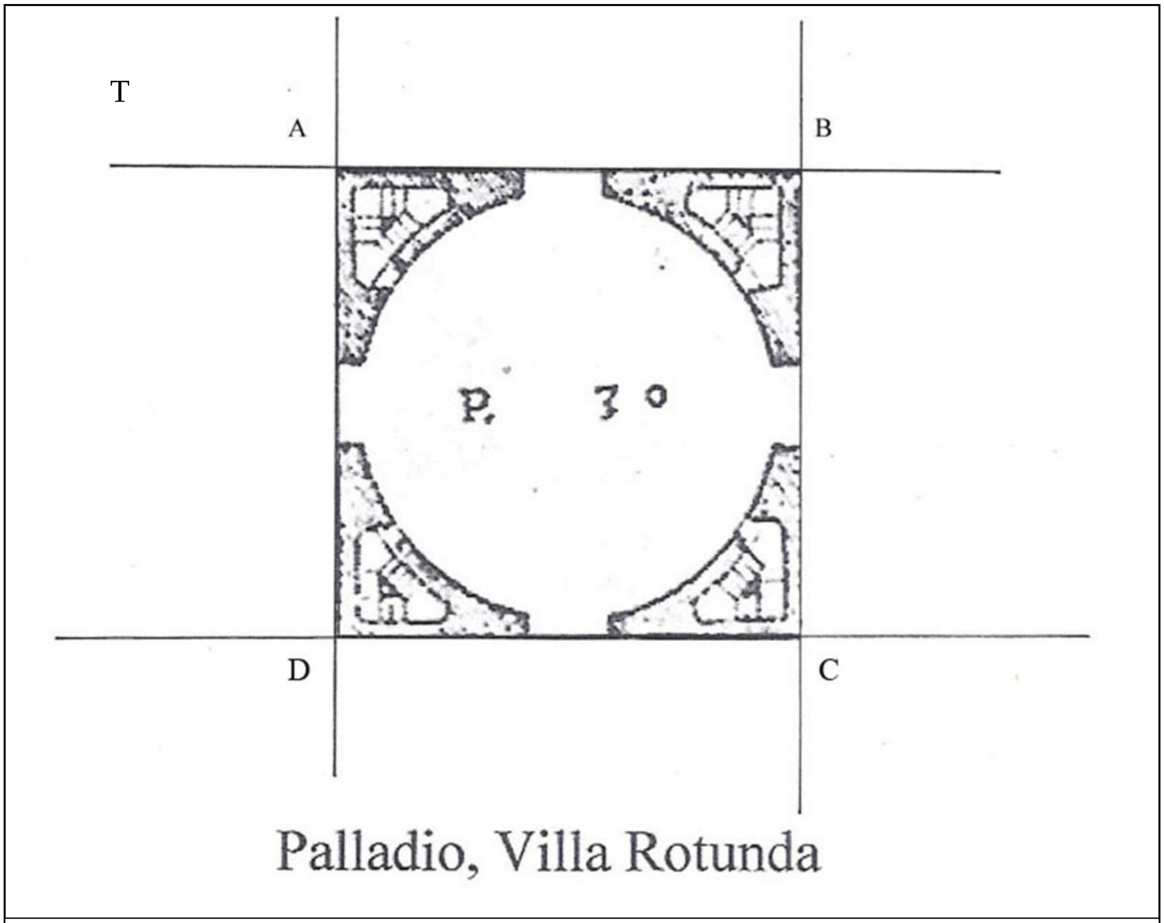


Fig. 64. The central base figure is designated square ABCD. Note the less obvious outer circle that is the circle circumscribed by the square.

The central circle in figure sixty four that is circumscribed by the square is actually the outer circular line seen more clearly in figure sixty five, rather than the more obvious inner circle. The circumscribing square is designated square ABCD and serves as the initial base figure for extracting the rest of the plan-net.

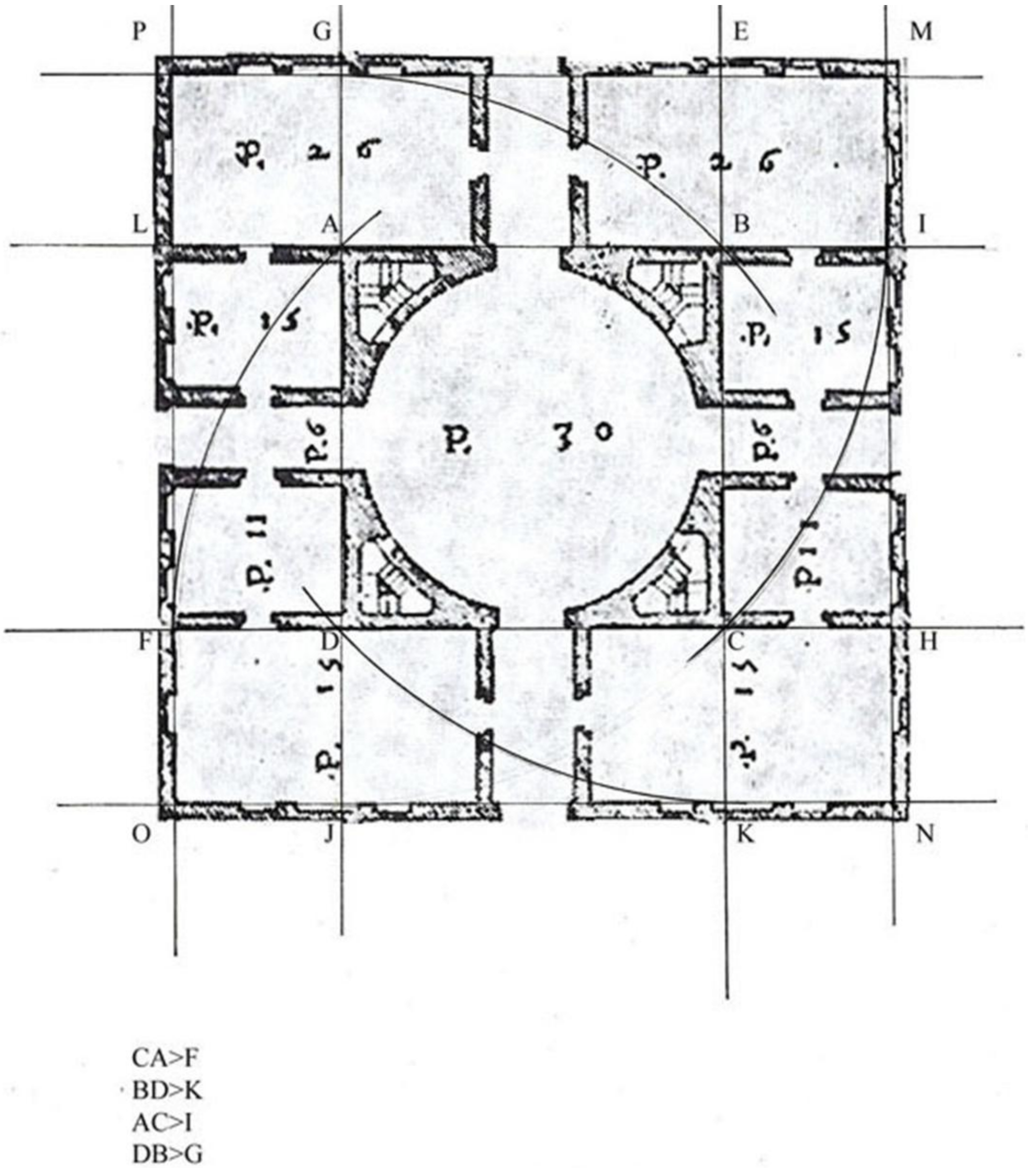


Fig. 65. The first set of extractions from the base figure.

In figure sixty five diagonal DB is rotated out to G, AC out to I, BD out to K and CA out to F. A larger square PMNO is formed because all of the diagonals came from the original square and were of the same length.

The next series of steps follows the same pattern but is not based on extractions from square PMNO. Instead, in figure sixty six it is the diagonals of four rectangles that are

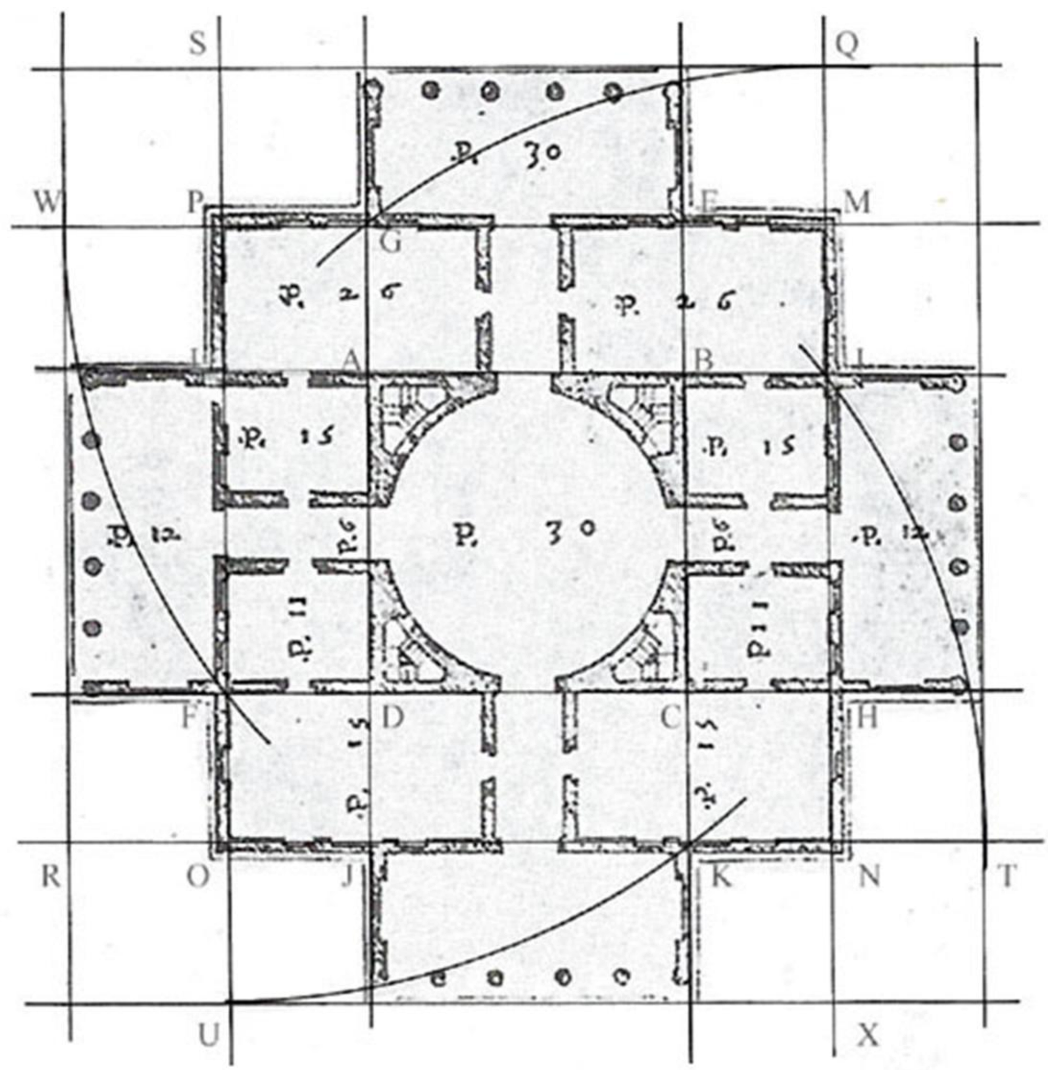


Fig. 66. Identical expansions add four new rectangles around square PMNO.

rotated, these diagonals being NG, OI, PK and MF. Diagonal NG of GMNJ is rotated to Q, OI is rotated to T, PK to U and MF to W. Because each of the rectangles used in this case are in the same proportions the diagonals are the same length, and so the newly created lines also join to form a square. However, the lines these four steps mark do not set out a new square, but rather, serve to mark four rectangles surrounding square PMNO as may be seen in figure sixty six. The lines marking the length of these rectangles are

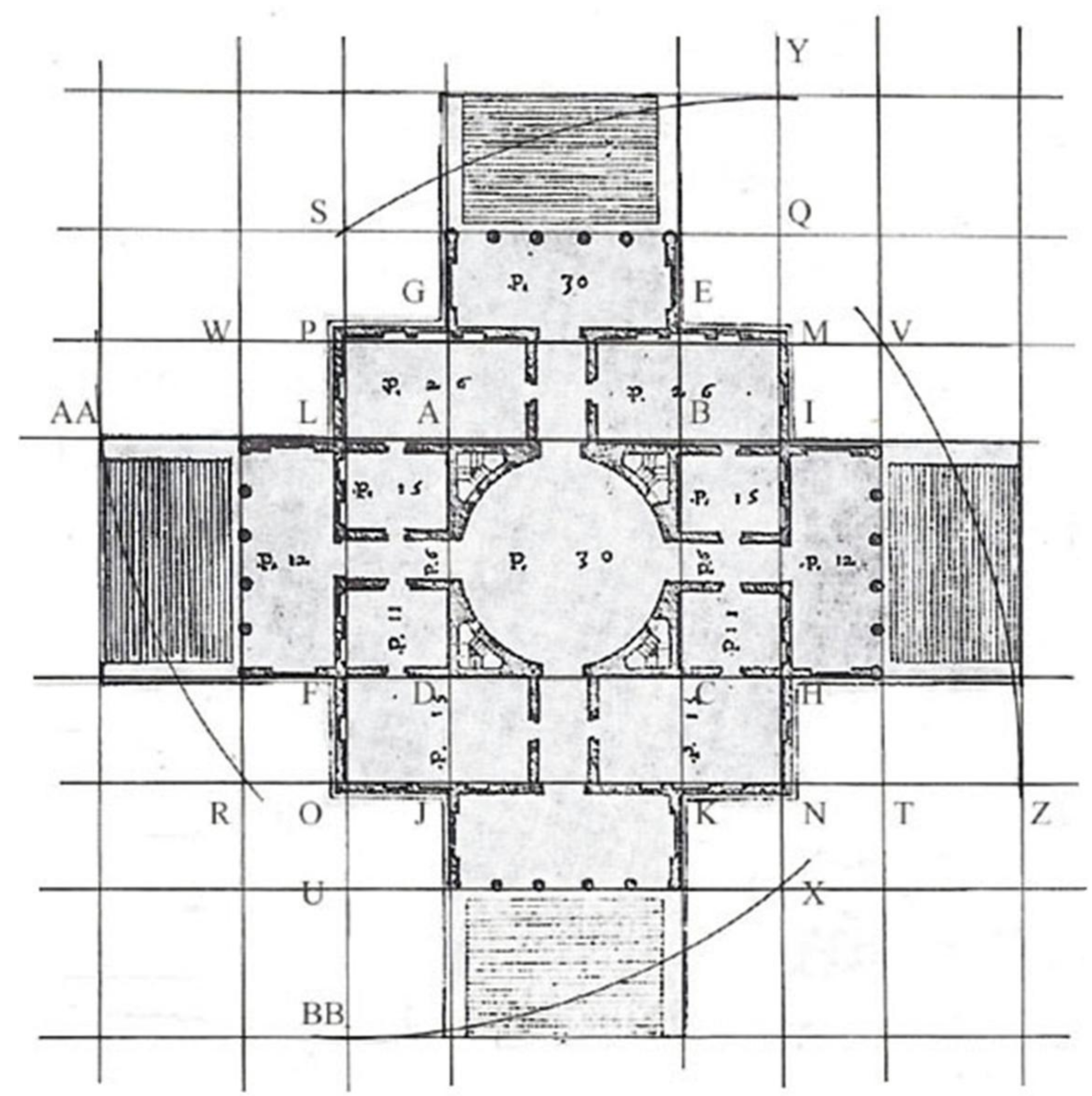


Fig. 67. A similar expansion with the diagonal of four rectangles completes the plan analysis.

lines LI, FH, EK and JG that contribute also as boundaries of the original square ABCD. The quadrilateral formed by these rectangular additions to the plan mark a larger square. In figure sixty seven a last set of expansions based on similar rectangles completes the structure, establishing its maximum width in all four directions. Diagonal NS is rotated up to Y and likewise the direction is reversed and PX is rotated to BB. Then OV is rotated to Z and its opposite MR is rotated to AA.

This analysis depends on similarly patterned steps taken in all four directions. The Bertolet plan and the Antes plan also call for expansionary steps taken in the four cardinal directions, but they are not all the same nor do they follow the same pattern. Bertolet consisted entirely of diagonal rotation steps, and Antes of diagonal rotation steps and an interval copying step. Palladio consists of diagonal rotation steps carried out in a consistent pattern at each expansion level around a central point. Bertolet and Antes rotate around a center but not in a consistent pattern. Richard Weiss alluded to this noting that "the hearth space lies in the middle of the living space... enclosed by dwelling, sleeping and work or storage areas on three outer sides of the house, like a mother encircled by her children."[90]

There is a great variety of ways in which a centered initial base figure can be expanded in all four directions depending on which diagonals are selected for rotation and how established intervals are copied to adjacent locations. Analyses that develop on a single axis we will call linear, monolateral if it develops in one direction only and bilateral if it develop in both directions. Analyses that develop on two perpendicular axes we will call rectilinear, bilateral if it develops on two directions, trilateral if in three directions and quadrilateral rectilinear if in all four directions.

That this pattern is present in structures as varied in background as the Bertolet and Antes houses and the Palladian Villa La Rotunda implies that the method of design

[90] Richard Weiss, *Hauser und Landschaften der Schweiz,* (Erlenbach-Zurich: Eugen Rentsch Verlag, 1959). 168

that produces the pattern is not associated with specific house types. The following analyses confirm this observation. All four may be termed quadrilateral rectilinear plans.

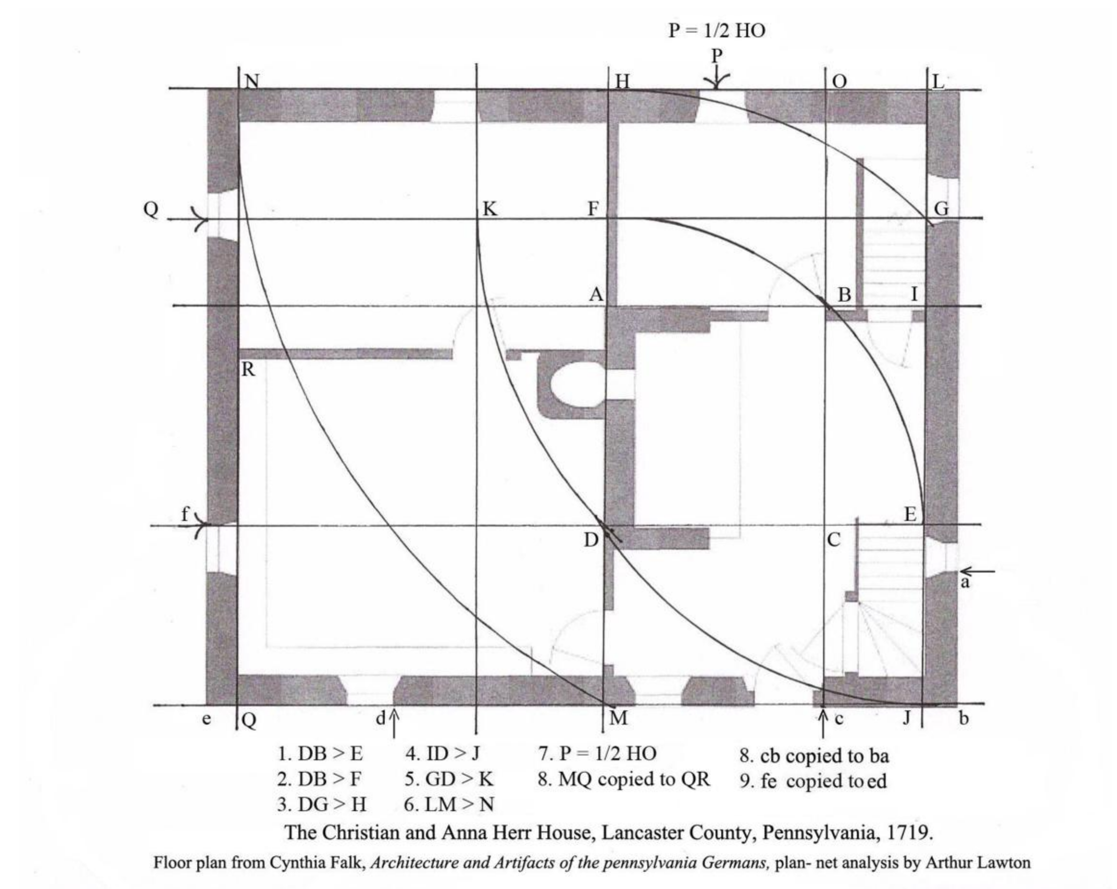


Fig. 68. Christian and Anna Herr House, Lancaster County, Pennsylvania, 1719.

The Herr House hearth is contained within initial square ABCD, one jamb inside and the other outside. One side of the reconstructed heating oven centers on side AD of the initial square. Diagonal DB rotated to E marks the right gable and to F marks a window at G. DG to H marks the rear wall, LD to J the front wall and LM to N the left wall. Windows are placed by copying intervals (de), (ef) and (cb). The window at P is ½ HO. The parlor partition is marked by copying MQ to QR.

The second house is the dwelling of Konrad Schultheiss in Eltersdorf, drawn by Rudolf Helm and demolished in 1799. With its additional steps it is a little more

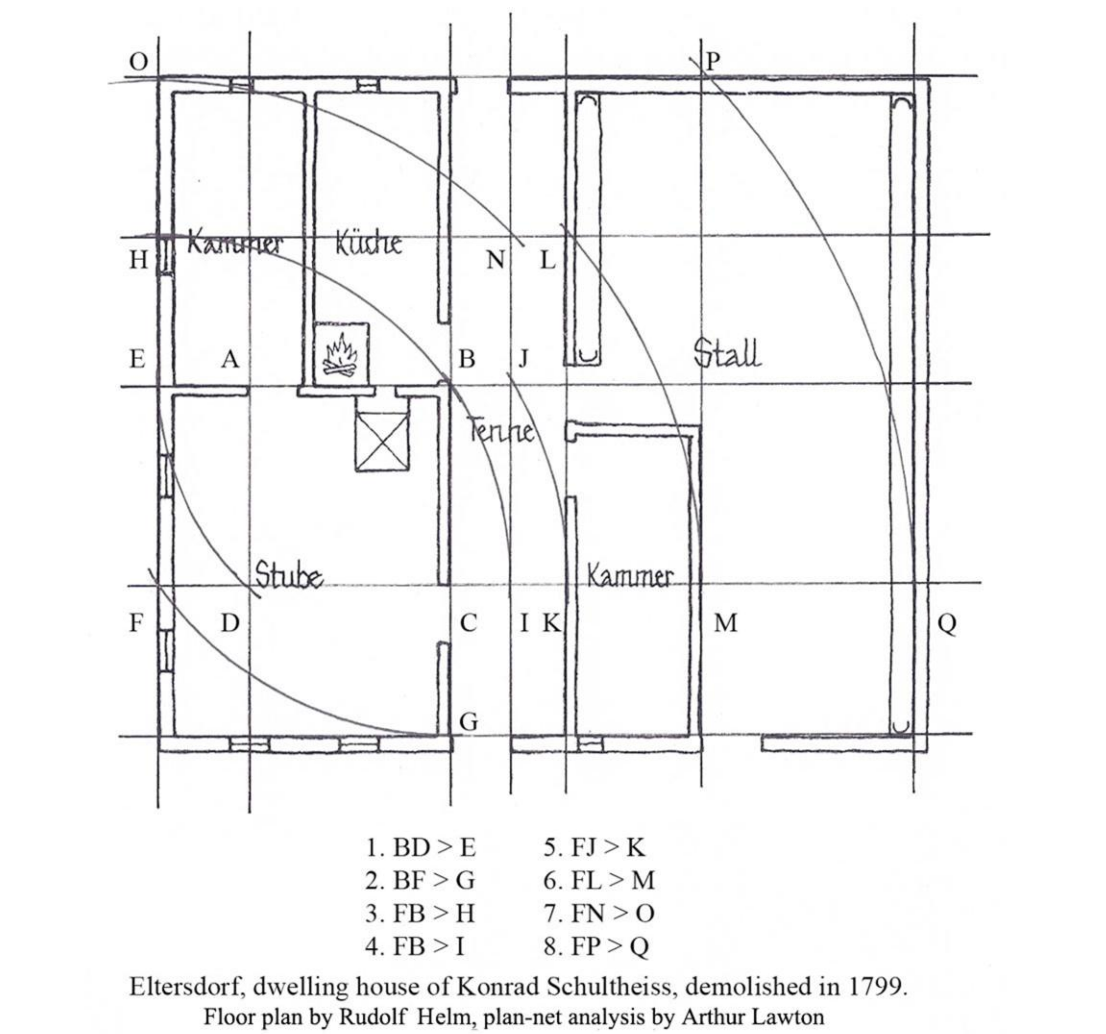
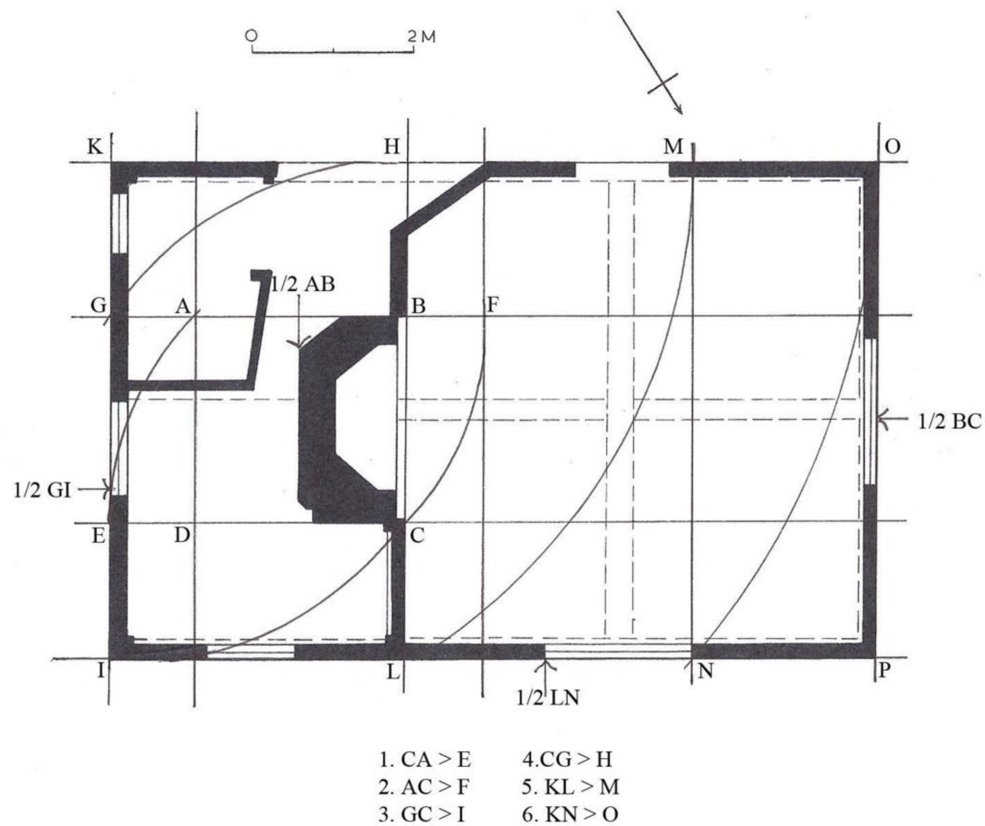


Fig. 69. Eltersdorf, dwelling of Konrad Schultheiss, demolished in 1799.

complex. The diagonal rotation lines are familiar from the previous analysis. The Herr house has six diagonal rotations, the Schultheiss house ten, to provide space divisions for both family dwelling and stall for the animals. Note also the essentially identical laying out procedure results here in an entirely different floor plan, called by Rudolf Helm a cruciform plan.

The third plan is that of a dwelling house at Church Farm, Great Waldingfield, Suffolk, England, drawn by Mathew Johnson in *Housing Culture* and that he says is



Church Farm, Great Waldingfield. Dated 1670 on projecting tie. Johnson, *Housing Culture*.
Floor plan by Matthew Johnson, plan-net analysis by Arthur Lawton

Fig. 70. Church Farm, Great Waldingfield, Dated 1670 on a projecting tie beam.

dated 1670 on a projecting tie beam. The initial square lies to the center of the house but is not precisely centered along the transverse axis and is considerably to the left of center on the lateral axis. The fireplace lies entirely within the square, the back wall of the fireplace on the ½ AB line and both jambs within the AB and DC lines. There is a single step CA to E to the left of square ABCD marking the left gable wall line, CG to H marks one lateral wall and GC to I marks the other lateral wall. The illustration does not make clear which side is the front of the house. Three steps to the right of square ABCD ending with diagonal KN to O that marks the right gable wall.

Rudolf Helm gives no date for the Kalchreuth stable of Heinrich Wittenschlager but it is certainly a sixteenth or seventeenth century building. These stables have little in

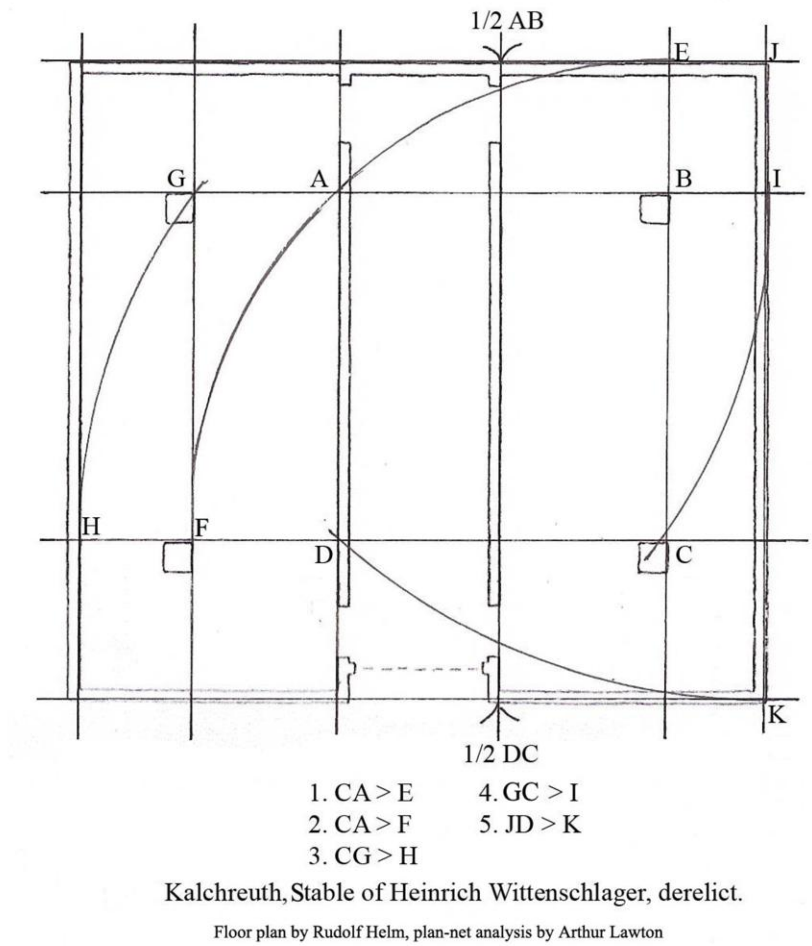


Fig. 71. Kalchreuth, the stable of Heinrich Wittenschlager, no date given.

the interior by way of space division other than the threshing floor and stalls. Therefore in the stable, the analytical lines relate primarily to the placement of the posts, outer walls and door. These buildings generally have four, six or eight posts. This one has four, the posts at B and C marked by the initial square. Side AD of the square marks the left side of the threshing floor and the door position. The right threshing floor side is marked at ½ AB. CA to E marks the rear wall and CA to F marks the line for the remaining two posts. GC to I marks the right wall and the final step JD to K marks the front wall.

These four analyses confirm the earlier observation that the design method producing a quadrilateral rectilinear pattern is not specifically associated with house type since we find common pattern in both dwellings and stables from varied time periods and cultural groups. The one building that stands out from these seven is the Palladian Villa La Rotunda. Its complete symmetry around a center point gives it a degree of formality not associated with the other buildings even though, like the other six its developing analysis moves in the same way, outward from the original base figure in all four directions. this latitude method allows development of either symmetry or asymmetry in the resulting plan, depending on how design steps are chosen.

Appearing very similar to plans designated quadrilateral rectilinear are those that are bilateral rectilinear. The difference lies in the fact that in quadrilateral plans the square base figure is centrally located in the fully developed plan. In the bilateral analysis the base figure must appear in corner of the plan, since the plan development appears in only two directions, but the plan being rectilinear, these directions must be along perpendicular axes. Since the base figure is located in a corner of the plan, all advancing steps of the plan development occur from the corner of the base figure that serves as the corner of the plan. A right or a left corner of the plan serves for this purpose. Thus it is not surprising to see that many floor plans can be analyzed as mirror images of other plans, from the right corner mirror imaging others developing from the left corner. Seven plans follow from the careful floor plans drawn by Rudolf Helm. Three house analyses develop from the left corner, three from the right and a similar distillery plan also from the right corner. These analyses show that a given plan development process can in fact be associated with a specific house type but is not limited to that house type. These seven plans show the greatest consistency of method of all groups examined. Some plans place the hearth on an exterior wall and some in the interior of the building. All show slightly different arrangements of space but the fact that all develop in the same way from a corner square shows the degree to which a plan development process can be adapted for accommodating individual preferences.

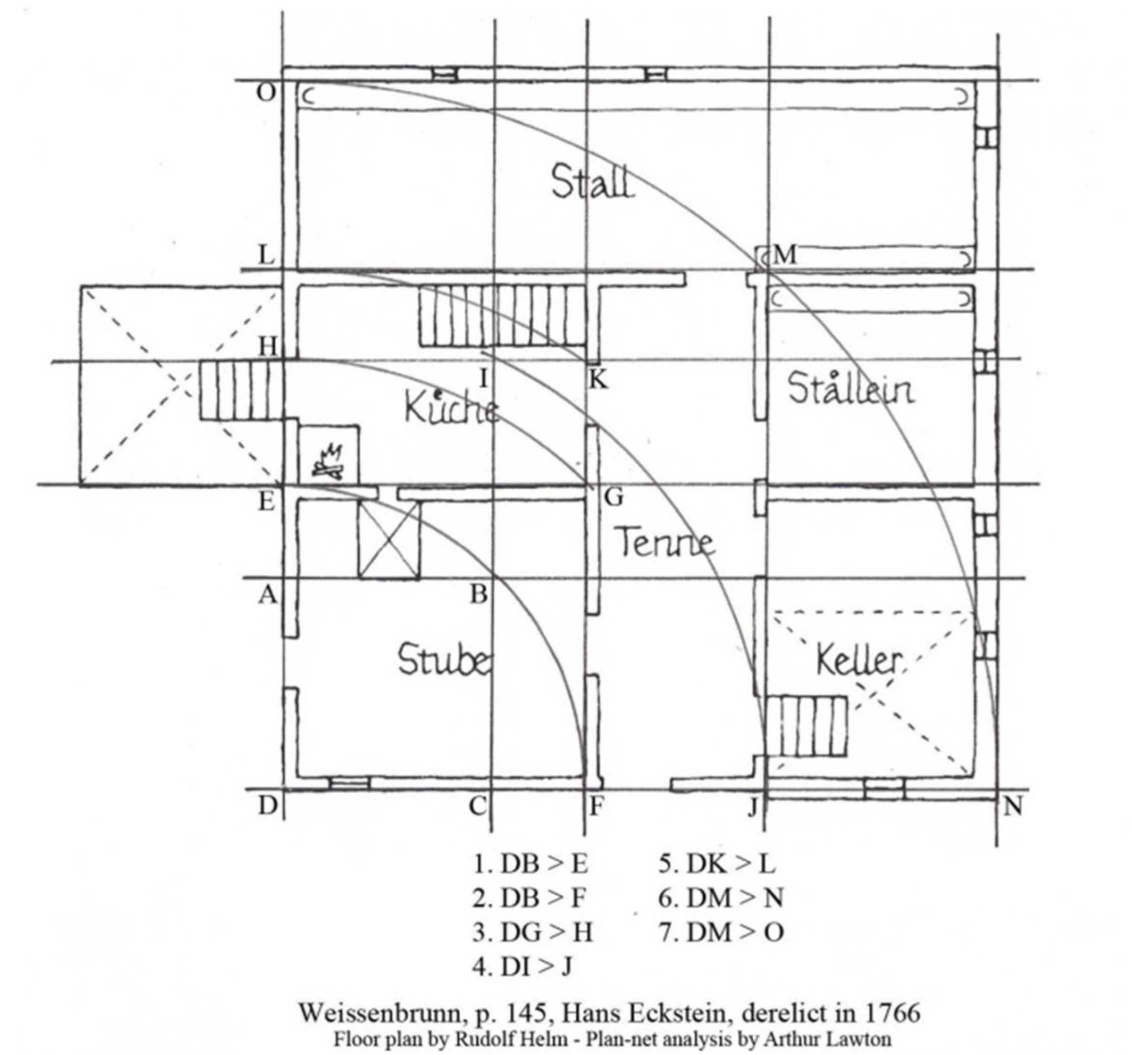
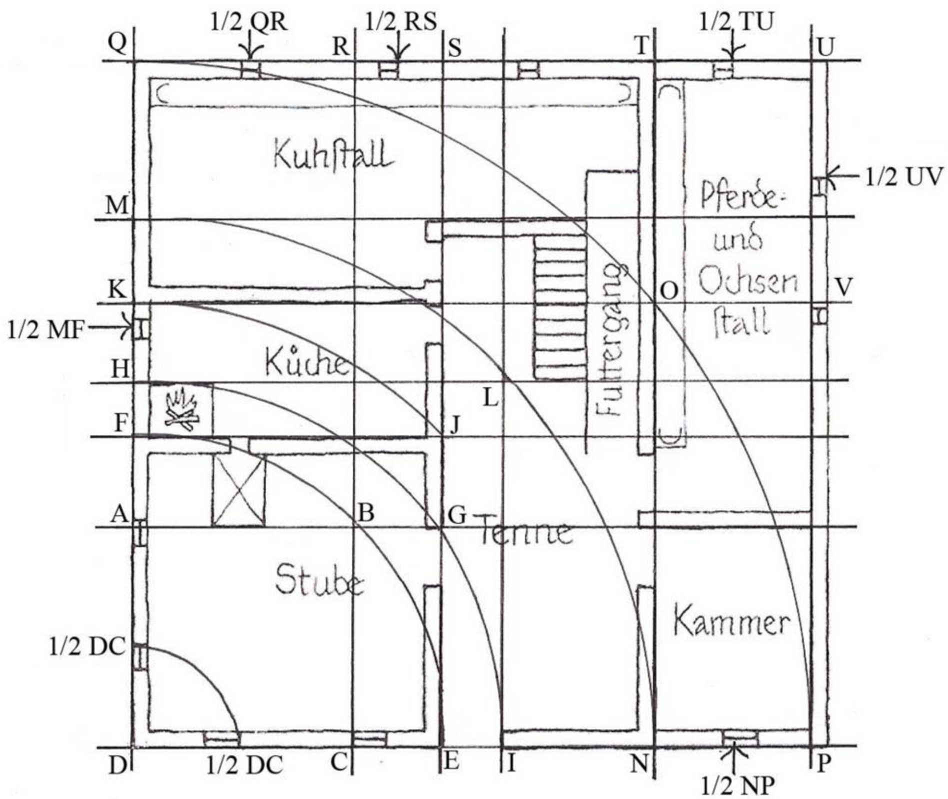


Fig.72. Weissenbrunn, dwelling of Hans Eckstein, derelict in 1766.

In Weissenbrunn, the half-timber house of Hans Eckstein was declared derelict in 1766. Its length was 36 feet and its width was 35 ½ feet, a length to width coefficient of .986. Helm describes this kind of floor plan as “strongly cross-shaped.” Page numbers in this set of illustrations refer to the illustration page in Rudolf Helms book *Das Bauernhaus im alt Nürnberg Gebiet.*



1. DB > E 4. DG > I 7. DL > N
2. DB > F 5. DJ > K 8. DO > P
3. DG > H 6. DL > M 9. DO > Q

p. 101. Haimendorf, dwelling of Paulus Sperber. Derelict in 1779.
-Floor plan by Rudolf Helm, Plan-net analysis by Arthur Lawton

Fig. 73. Haimendorf, dwelling of Paulus Sperber, derelict in 1799.

The Haimendorf dwelling house of Paulus Sperber, derelict in 1799, was 38 ½ feet long and 39 ½ feet wide, a coefficient of .974. Like the Eckstein dwelling in figure 64, the hearth is located along the left gable wall and the parlor heating oven more to the interior. While the room and stall positions are the same, the proportions differ. Helm's reference above to "strongly cross-shaped" refers to the kitchen (*Küche*) on the lateral

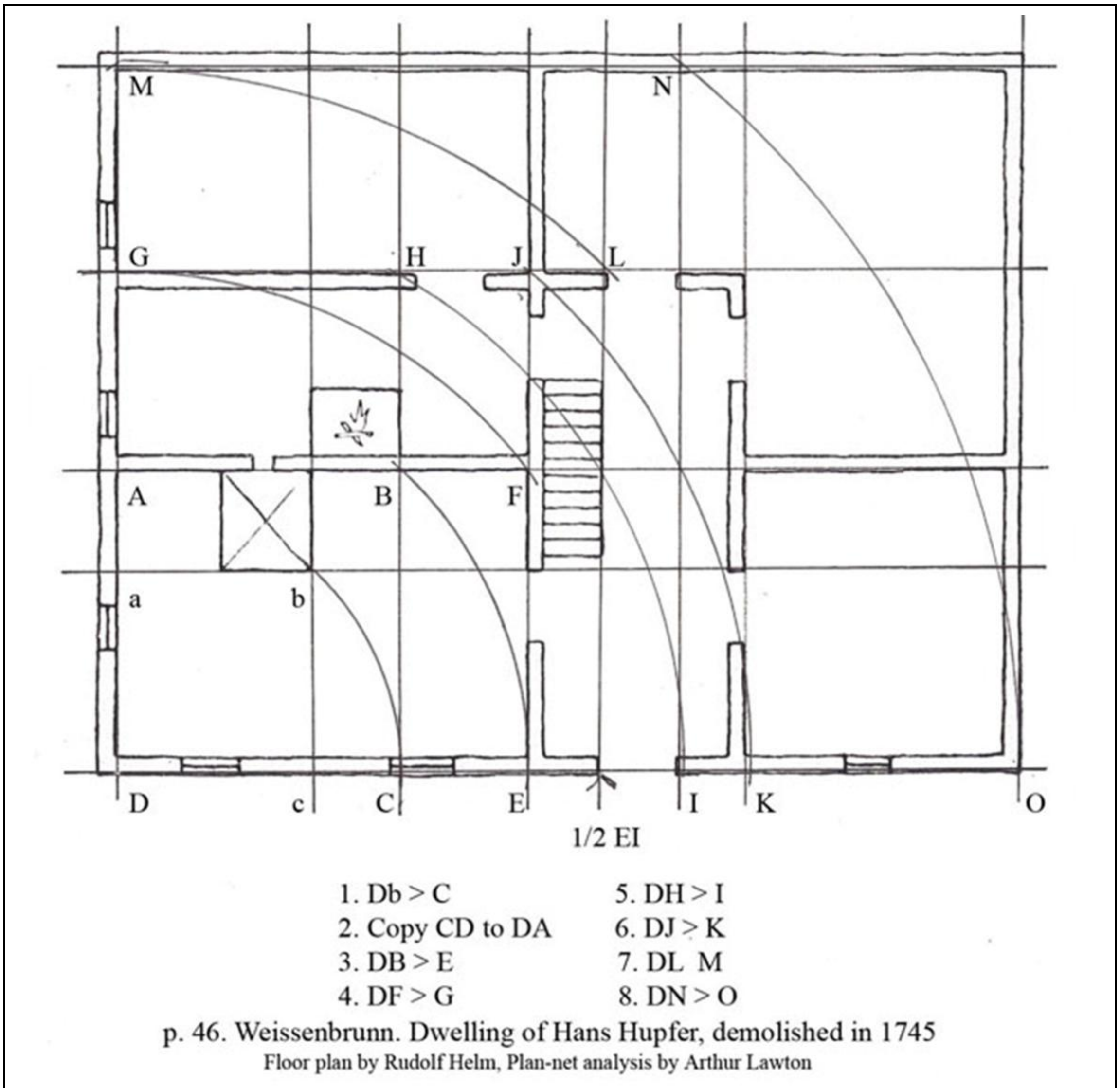


Fig. 74. Weissenbrunn, dwelling of Hans Hupfer, demolished in 1745

axis and the *Tenne* (threshing floor or through hallway) on the transverse axis.

The Weissenbrunn dwelling house of Hans Hupfer was demolished in 1745. It was 41 feet long and 32 feet wide, a coefficient of .780. The coefficient of .780 is very low and thus considerably longer than wide. Note that though the overall arrangement is very much the same, the hearth is located on the other side of the heating oven and is well to the interior of the plan.

When plan development shifts from left front to right front corner, the result is for all practical purposes a mirror image. The following four plans, in terms of their design

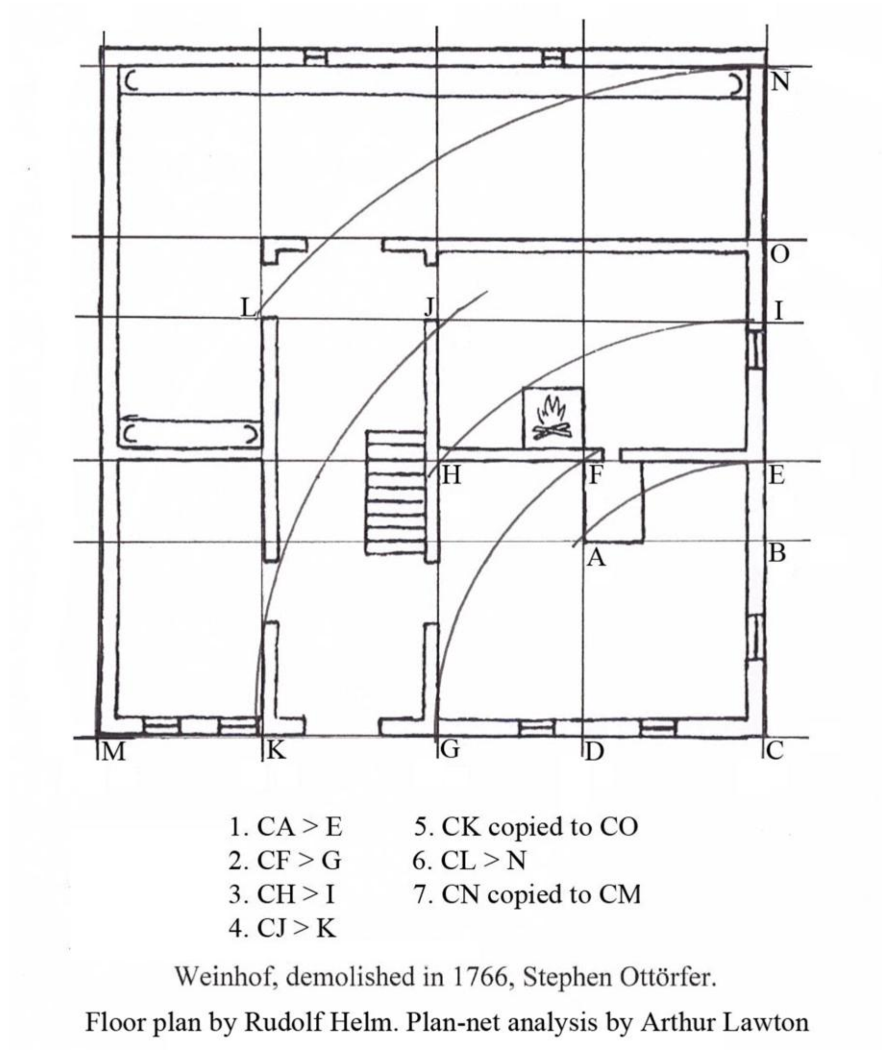


Fig. 75. Weinhof, dwelling of Stephen Ottörfer, derelict in 1766.

method, are mirror images of the previous five. Their plan-net analyses are generated from the right front corner by follow the same pattern of development.

The Weinhof dwelling of Stephen Ottörfer, demolished in 1766 was 33 feet long and 34 feet wide, a coefficient of .970. The hearth and heating oven are not on the gable but rather more centrally located.

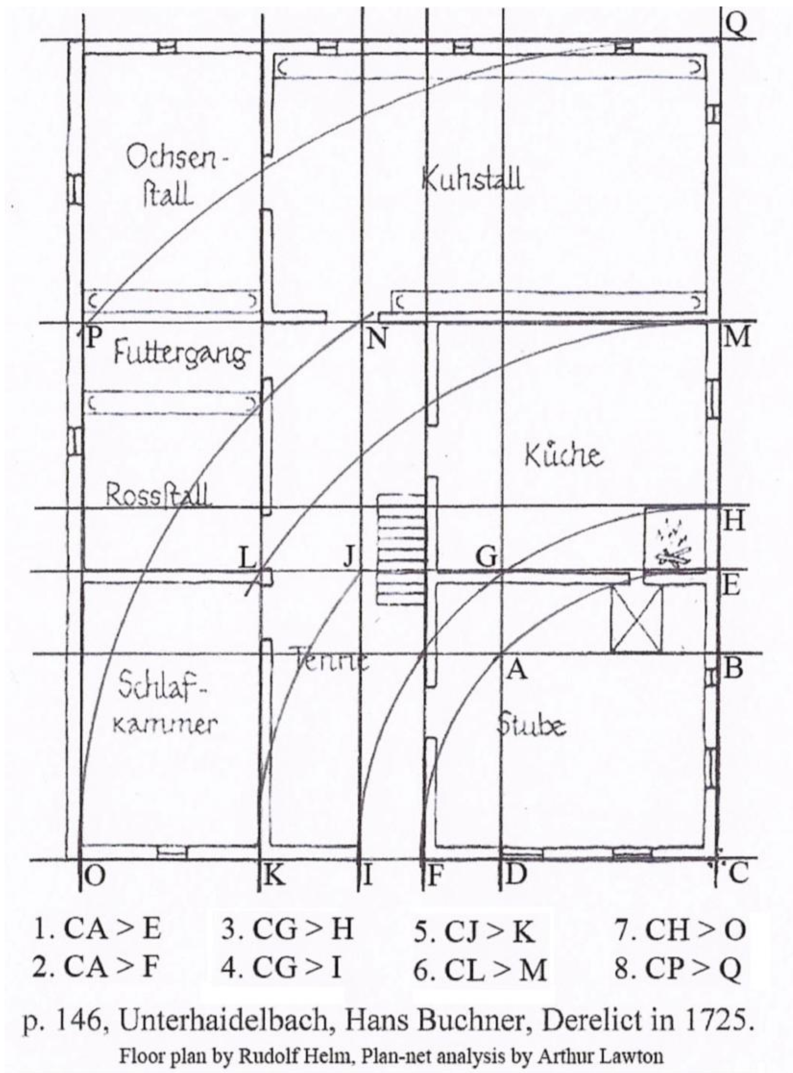


Fig. 76. Unterhaidelbach, dwelling of Hans Buchner, derelict in 1725

The dwelling house of Hans buchner in Unterhaidelbach, derelict in 1725 was 50 ½ feet long and 40 ½ feet wide, a coefficient of .801. Helm describes this proportion as *ungefahr*, casual, rough, apparently meaning only roughly conformable to expectation.

The plan itself differs only in minor details and in the proportional arrangement of internal space. However, the low length to width coefficient is unusual in that the plan is significantly less square than expected by tradition. Note that it is the heating oven rather than the hearth that is adjacent to the square base figure.

The Entenberg dwelling of Georg Heberlein was 32 ½ feet long and 28 feet wide, a coefficient of .861. It was derelict in 1745. Helm notes that this house has three

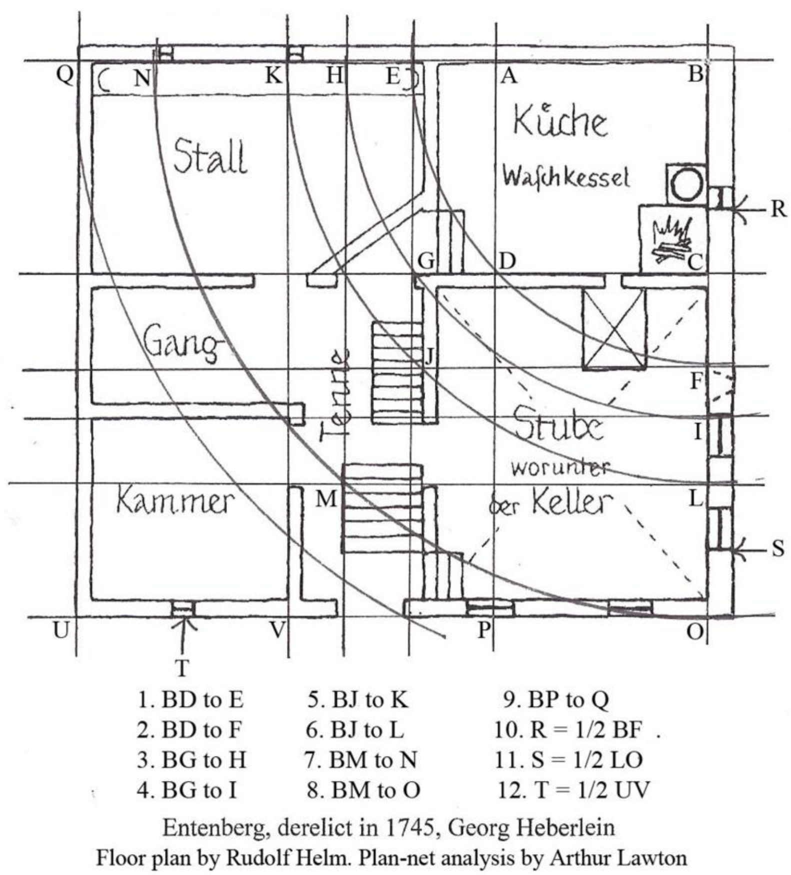


Fig. 77. Entenberg, dwelling of Georg Heberlein, derelict in 1745.

different construction processes. Principally a half-timber house, on the stable side “the half-timbering is partially filled with log construction in place of the expected mud mortar and on the dwelling side the outer wall is constructed in stone. The *Stube* has proper windows, the *Kammer* only a very modest one and the *Küche* only light openings.” Helm says, “The house developed not by adding on, but as a harmonious and well-formed whole.” That it submits so neatly to plan-net analysis also suggests that the house was a harmonious and well-formed whole. The use of varied building materials points more likely to the rigid limitations placed by the forest bureau on timber available from the forest than to construction at different periods of time.

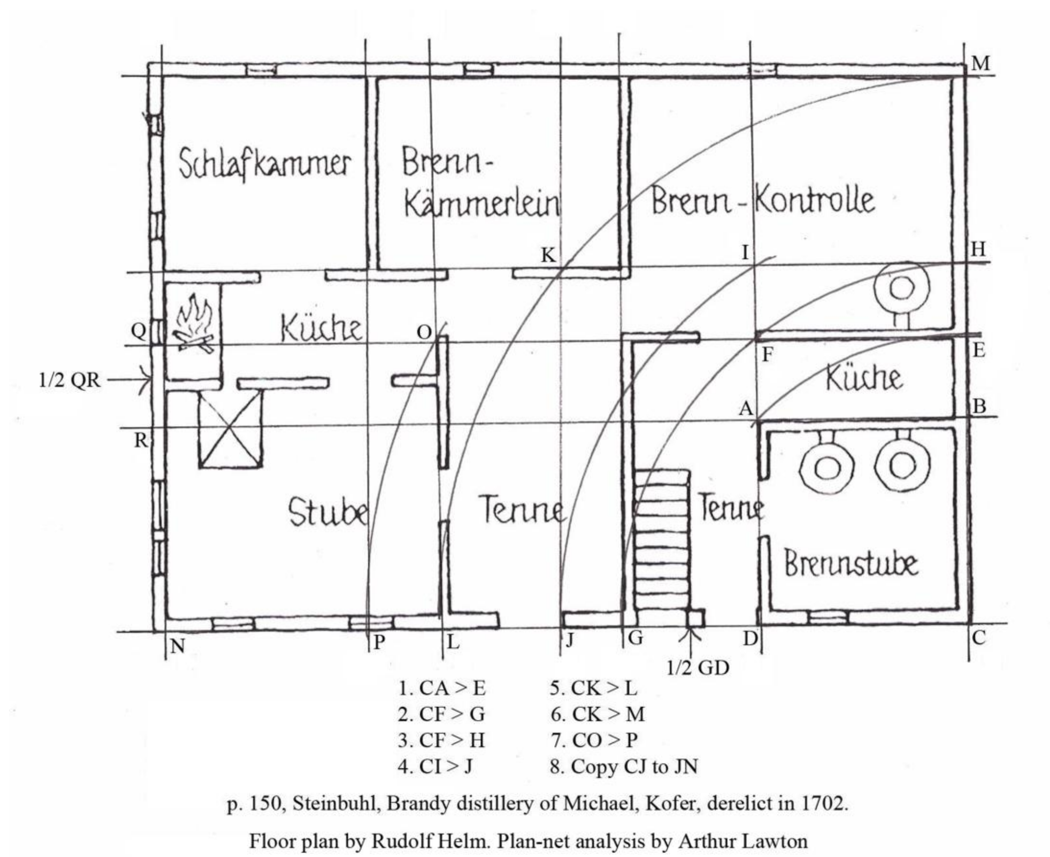


Fig.78. Steinbuhl, Brandy distillery of Michael Kofer, derelict in 1702.

The brandy distillery of Michael Köfer at Steinbühl, declared derelict in 1702, was 39 feet long, twenty seven feet wide for a length to width coefficient of .692. Helm notes that this does not disavow its origin as a traditional dwelling house. It follows the traditional floor plan with the distillery in place of the stalls. With the lowest length to width coefficient of all, it is of interest to note that the length is accounted for by the fact that just as did Henry Antes, a lateral interval is doubled, in this case CJ is copied to JN. Also to be noted is that the square base figure in this case is marks the location of two of the three distilling kettles, these being perhaps closer to the heart of the distiller than the cooking hearth.

The early Anglican churches studied by Dell Upton in *Holy Things and Profane* meet the eye as linear buildings and so they are in a very real sense, though in the terminology of plan-net analysis they are otherwise. To be analyzed as a mono- or bilateral linear building, the initial base figure must be the full width of the gable, with no possibility of transverse development of the plan. In these churches the base figure appears in a corner of the structure as did the bilateral rectilinear dwellings just examined. The one or two transverse steps taken to establish the building width make the plan rectilinear, but all remaining development occurs along the lateral axis in a single direction.

Upton, writing about St. Luke's Church, constructed ca. 1685 in Newport Parish, Isle of Wight County, Virginia notes that "all the elements of the form to be retained by these churches through the time that the Establishment was dissolved, were in place" by the time St. Luke's was constructed. He adds that "the Virginia church belongs to a tradition of church planning and design established in the early seventeenth century."

He noted that the needs required for Anglican Church functionality in Pre-modern Virginia were highly unified across time and across space. That is to say, the role of all the participants during the community's liturgical activities were carefully prescribed, well understood and universally practiced. He points out that there were three broad types of church building, the rectangular, the cruciform and the deep building. [91]

It is not surprising then to find that plan-net analysis reveals a similar uniformity of pattern in the development of these church building plans of a single building type. This occurs not simply because the activities carried out with the building are highly uniform, but at a deeper level because plan-net analysis demonstrates that builders followed essentially the same pattern in the process of designing. We see a remarkable uniformity of pattern not only in the resulting physical structure of the Early-Modern Anglican Church building in Virginia as well as in the subsequent use of the building, all indicative of a secure and stable tradition, and yet as will be seen in the plans there is also some degree of variation suggestive of individuality specific to location and designer.

As in the case of vernacular houses, the analysis process begins by locating the most symbolically significant element of the plan. In the vernacular Bertolet house, it is the hearth. In the Ancient Greek Villa of Good Fortune excavated at Olynthos it is a small square at the center of the atrium, probable an altar or herm location and in the ancient Hittite fortress palace at Boghazkoi it is the courtyard where the ruling elite met the public. In the Anglican Church of the late seventeenth century it is the location for the communion table at the liturgical east end of the church. Hearth and communion table are not necessarily encompassed by the initial square, but in their position they are related in some way and at least adjacent to the square.

We begin with Mangohick Church in King William County, built circa 1731. In addition we will examine three other floor plans in this collection, all dating no later than the first third of the eighteenth century. These churches are Middle (Christ) Church

[91] Dell Upton. *Holy Things and Profane*. (Cambridge: The MIT Press, 1986). 59.

(begun 1712), St. Peter's church (1701-1703), and St. Luke's Church, constructed circa 1685 in Newport Parish, Isle of Wight County.

In figure seventy nine square ABCD is the base figure for the Mangohick plan located in the right hand corner. In Episcopal liturgical practice, the altar end of the

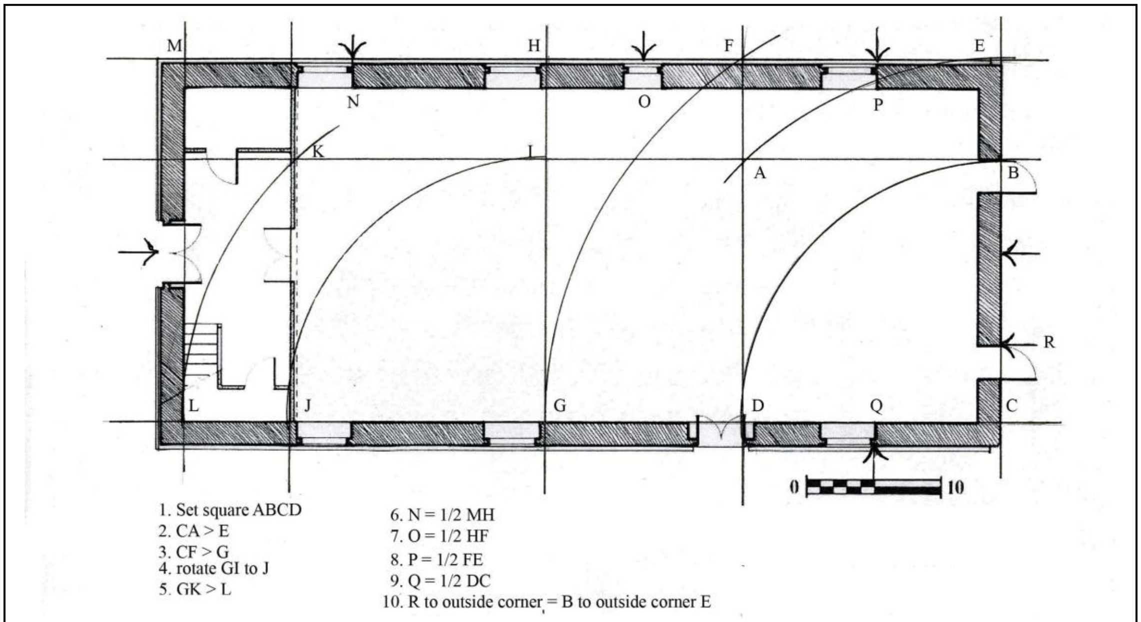


Fig. 79 Mangohick Church.One transverse and three lateral steps complete the plan.

building is liturgically the east end. The gable wall is placed inside the square and the right lateral wall is outside the square. Though the altar position is not indicated on Upton's floor plan, it will be centered on the gable wall, placing it inside square ABCD. The first step advances on the transverse axis to mark the location of the left side wall inside line ME and thus determining the width of the building. Unlike the Antes houses, this expansion is limited to one step.

The direction then changes to the lateral axis where it will remain for the rest of the analysis. Diagonal CF rotated down to G, marks line GH and the window pair at G and H. The next step copies line segment GI, a part of line GH, down to J creating a square identical to square ABCD. The last step rotates diagonal GK to L to mark the liturgical west gable of the church. The position of window N = ½ MH, of window O = ½ HF, of window P = ½ FE and of window Q = ½ DC. The other east gable door is marked at the interval ER by copying the interval from the outside corner by C to the existing door at B over to the other corner from E to R, preserving gable symmetry in placement of the doors.

In figure eighty Middle Church is also known as Christ Church and the initial square base figure is in the corner on the right side of the liturgical east end of the

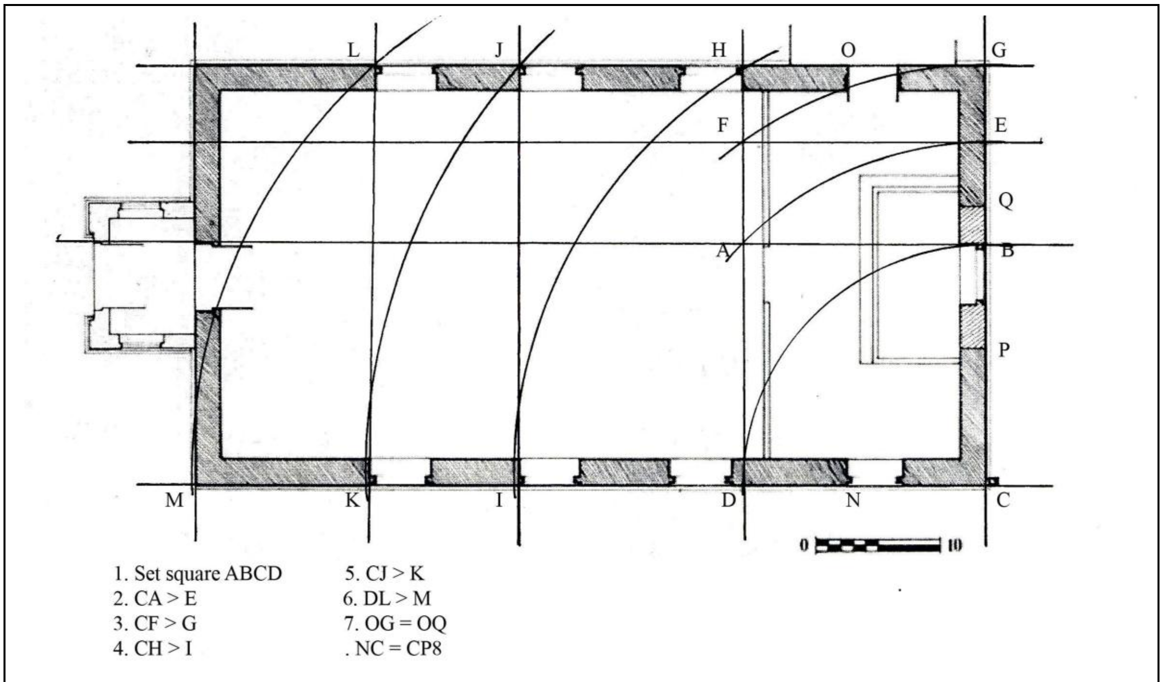


Fig. 80. Middle Church. Two transverse expansions determine the building width.

building. Here the square is much smaller in relation to the overall size of the building. The first expansive step rotates diagonal CA to E and the second step rotates diagonal CF to G to mark the left hand gable corner at the east end.

As will be the case in all of these church plans the first lateral step utilizes the full gable width of the plan. Diagonal CH is rotated downward to mark a pair of windows on line IJ. In all four plans this first lateral step marks either a pair of windows or a pair of buttresses. The second lateral expansion, diagonal CJ rotated down to K, marks windows at K and L. All diagonals up to this point have been taken from corner C. The final lateral step pivots not from C but from D. It is not at all unusual for such advancement of the pivot point, especially in an exceptionally long structure, as may be seen in Welsh long houses. Diagonal DL is rotated downward to mark the west gable wall at M. Windows at O and N are marked by copying QG to GO and PC to CN. The entrance way at the left end of the plan is not included in the analysis as it was a later addition to the building. The analysis of Middle Church achieves much the same ends as that of Mangohick Church, but does so entirely by diagonal rotations. There are no copying steps as in Mangohick.

The following church, St. Peter's seen in figure eighty one, also has an entrance tower that is not original as well as a somewhat different development process. Side AB marks the edge of a centered gable window at B and a single transverse step, CA to E marks the width of the building. CA is again rotated, this time down to G to mark a door at what is probably the chancel rail location. Diagonal CH is then rotated down to I to mark a pair of windows at I and J. Analysis shows a different procedure from this point on, interval copying rather than diagonal rotation.

Point H is the pivot point by which line segment EH is copied over to HL, marking line segment LK, upon which a window pair is found at L and K. Similarly, point J is the second pivot point by which line segment EJ is copied to JM, marking line MN and the gable to gable length of the building proper. As previously noted, the entrance tower is a later addition. Of interest is the fact that, beginning at the east gable, intervals CD, DL and LK are equal, providing windows that within the limits of visual tolerances are relatively equally spaced. However, KN to the west gable is longer than the other three. The rationalfor this is be explained by the plan-net analysis lines as follows.

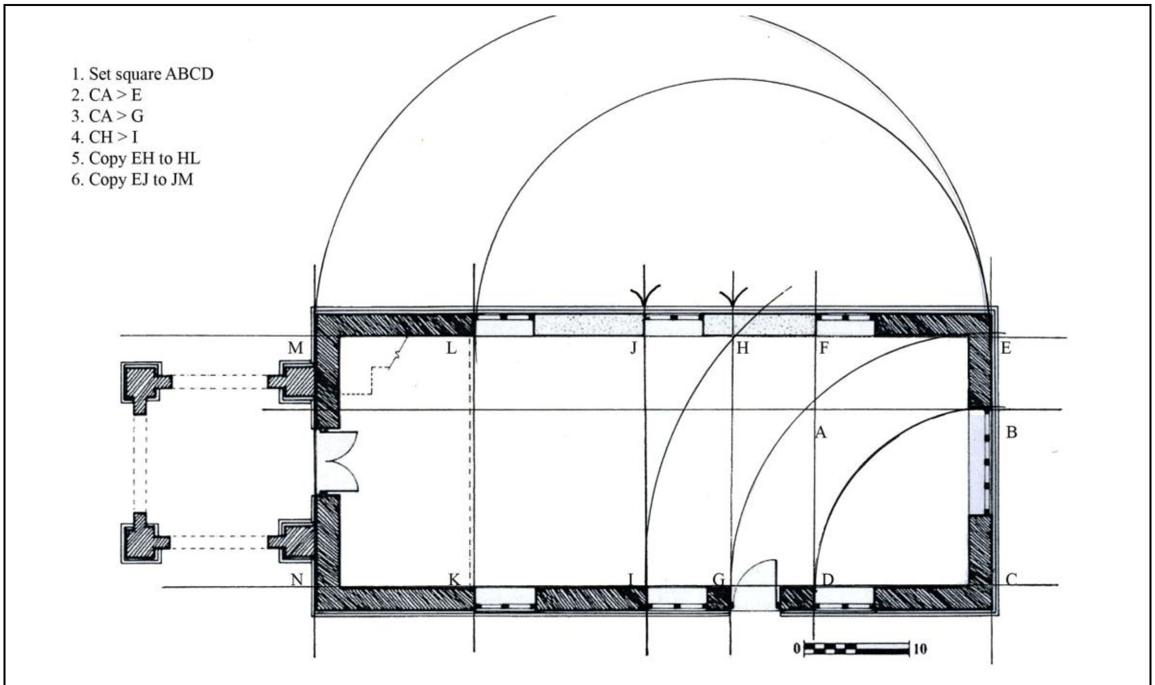


Fig. 81. St. Peter's Church. The plan is completed with two copying steps from center point J.

ABCD is a square as is HECG. If, as is indicated by the scale, DC = 17 feet, by the Pythagorean theorem and using diagonals CA and CH, CG = 17 x √2 = 24.041 feet and CL = 24.041 x √2 = 33.999 feet. CG copied to CK = 24.041 x 2 = 48.083. Windows at D and L and the door at G are thus marked at 17, 24 and 34 feet by plan-net lines within tolerances of hundredths of a foot. The window at K is at 48.083 feet, again within tolerances of hundredths of a foot. If CL is 34 feet, then CN is double that, or 68 feet. NK then equals 68 feet less CK equal to 48 feet, leaving about 20 feet, noticeably longer than the interval betwwen the other windows. Falling on the lines as they do, the windows are nevertheless centered sixteen feet apart. The distance of the right gable to the nearest window edge is 11 feet while the distance to the left gable is 15 feet. The determinative factor may well have been space arrangements that were dictated by the intended use of interior space. However even though all the above measurements approximate within visual tolerances the whole unit values of 11, 15, 17, 24, 34 and 48 feet, the means of implimenting the arrangement can just as well be explained by geometrical steps rather than modular measurement.

The plan of St. Luke's church in figure eighty two demonstrates three means available for use in developing the full length of the bulding. The analysis begins as in

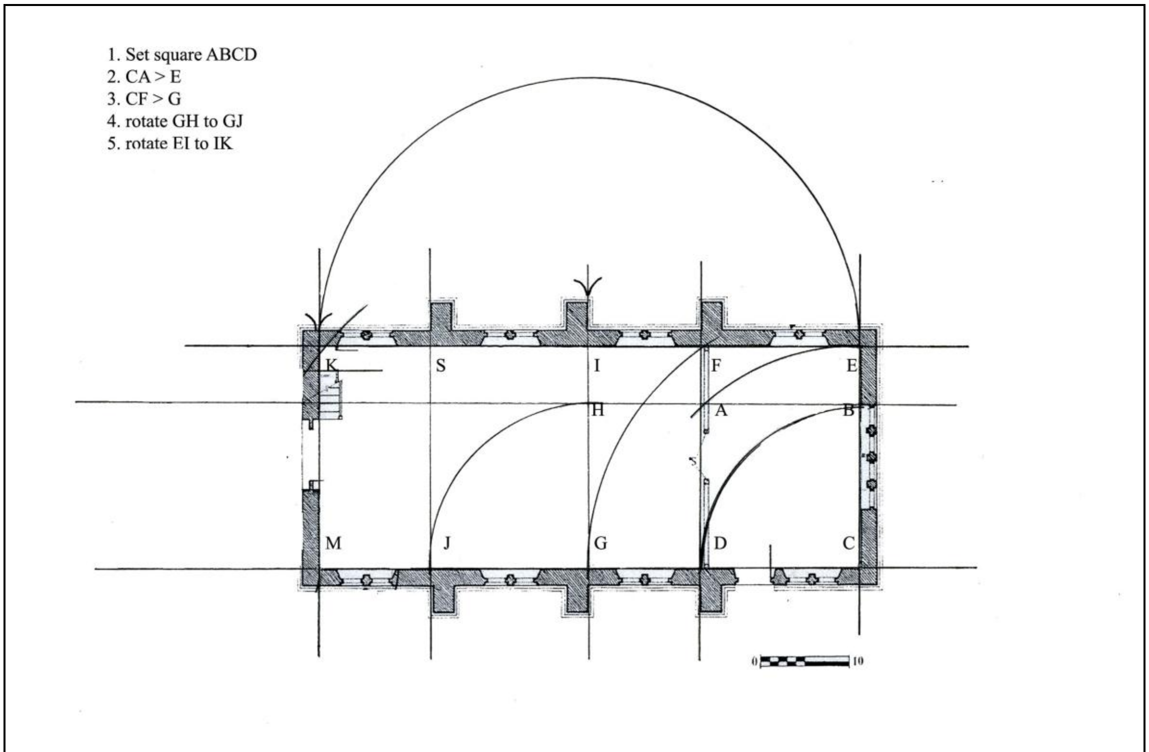


Fig. 82. St. Luke's Church. The sanctuary structure is completed with a single copying step rotated around center point I.

the other three churches by developing first the width of the building. The initial square ABCD marks the first pair of buttresses at D, the chancel rail along side AD and the edge of a window centered on the gable wall. The first steps are by diagonal rotation. Diagonal CA is rotated up to E marking the full width of the building and then diagonal CF is rotated down to G to mark the location of the second pair of buttresses.

The next step copies line segment GH around the corner at intersection node G to GI. This establishes a second square marked by JGH that is the duplicate of the initial square ABCD. Side SJ of this square marks the position of the third busttress pair. To complete the main building plan, line segment EI is copied around its end point at I to mark IK for the other gable wall at K. The plan of the main building is now complete. Like the windows of St. Peter’s Church, though all windows and buttresses are equally spaced in relation to one another, there is a difference at each end in the distance to the corner of the building. It can be argued however that the windows and buttresses are related to the lines of the analysis as a set, for the analytical lines do explain the position

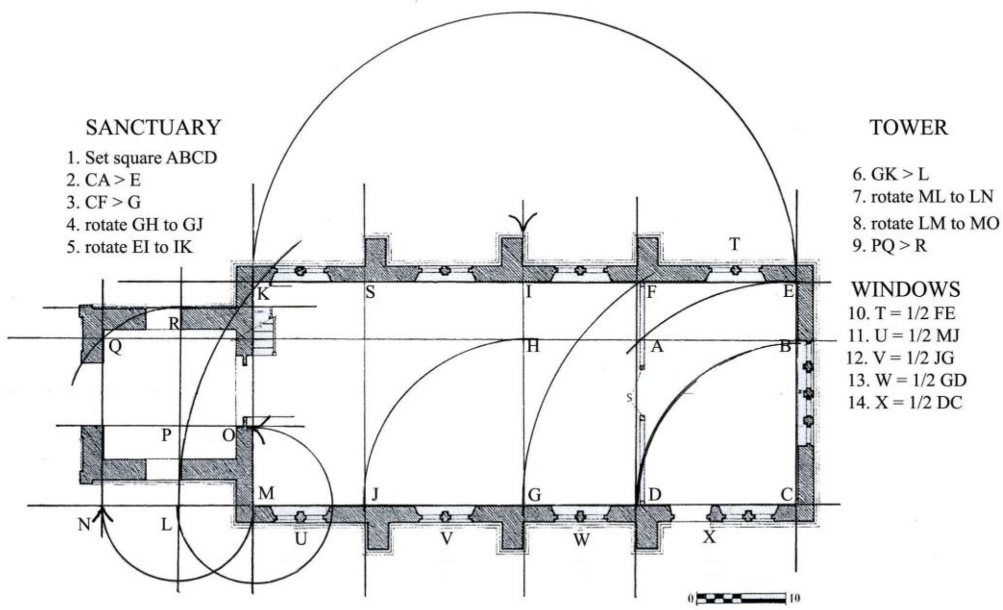


Fig. 83. St. Luke’s Church. The tower analysis relates the tower to the sanctuary plan.

of the windows and buttresses as a unit in terms of the plan-net. To the extent that the analysis accounts for this difference the concept of plan-net analysis is validated.

It remains now to account for the tower at the entrance to the building. How does it relate to the plan-net structure of the sanctuary structure? In figure eighty three diagonal GK is rotated down to L marking line LR. LM is then copied around the corner at M to mark the front sanctuary doorway at O. This establishes node P on line LR. ML is then copied to N, marking the facade of the tower and establishing an intersection node at Q. The final step is to rotate diagonal PQ to R marking the other side if the tower.

We have seen thus far that quadrilateral rectilinear plans begin with a square base figure that is located toward the central area of the completed plan. Advancement of the plan-net from that square occurs in all four directions. The bilateral rectilinear building begins with a square base figure located in one corner of the plan and advancement occurs in one direction only on two perpendicular axes. Both strategies give plans whose squareness coefficient approaches 1.0 as squares or rectangles, but that can also be longer rectangles as in the above churches.

Where a much lower squareness coefficient is customary, trilateral rectilinear buildings begin with a square base figure that is located along one wall, rather than in a corner of the plan. This allows for advancement along a lateral axis in two opposite directions and advancement on a transverse axis in one direction. Because the squareness coefficient of these buildings is so low, it is tempting to call them linear, but if one holds to the terminology of plan-net analysis, they advance along perpendicular axes and so must be considered as rectilinear.

We begin with Ellen Cutler's dwelling in Ballymenone, Ireland for which the plan was drawn by Henry Glassie and published in *The Stars of Ballymenone.* In figure eighty four we see the square base figure ABCD is located along the front wall toward the left end of the plan. The third direction that makes it trilateral functions to somewhat widen the building. The hearth and chimney stack are located adjacent to the B corner of

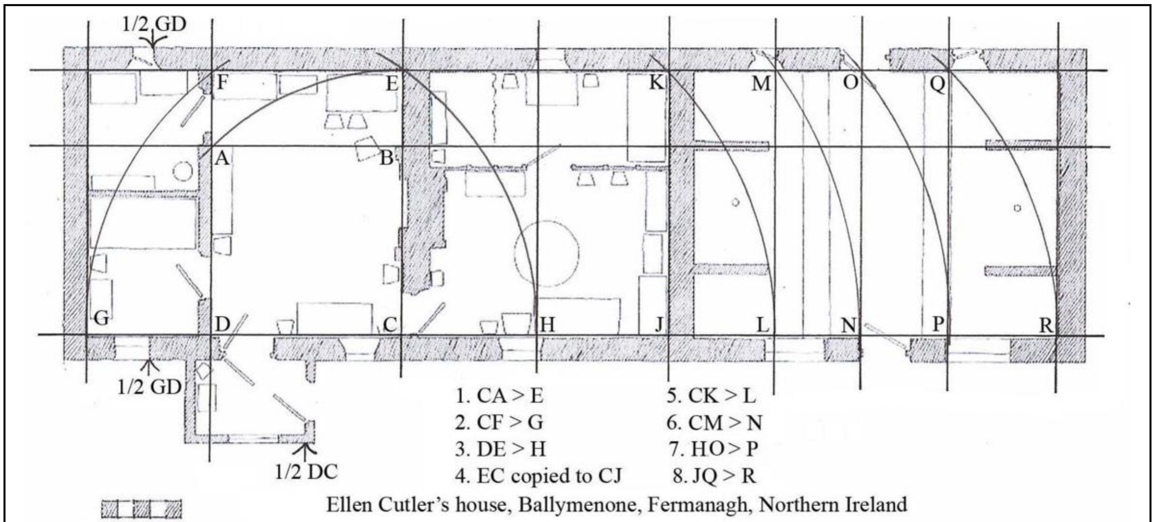


Fig. 84. Ellen Cutler's dwelling. Plan by Henry Glassie, from *The Stars of Ballymenone*.

the original square on its right (BC) side. The gable width of the house is gained by rotating CA to E and diagonal CF of FECD is rotated to G to mark the left gable. DE to H marks two windows on the line at H and copying EC to CJ marks a square containing the parlor. The linear plan developed thus far is then extended to the right in a series of steps that advances the pivot point in the sequence first from D to C as CK to L and CM to N, then by advancing the pivot point to H, rotating HO to P and then advancing the pivot point again to J and rotating JQ to R. This strategy of advancing the pivot point is found in the analysis of all houses examined whose lateral axis is the defining proportional characteristic of the plan.

Where in the Cutler house the square base figure accounts for three quarters of the gable width, in the house in figure eighty five at Clooney Townland, North County, Londonderry, Ireland, the square base figure accounts for only about one third of the gable width. Clooney is exceptional in this group for the number of steps taken on the transverse axis to achieve the gable width. The hearth is centered on corner D of the

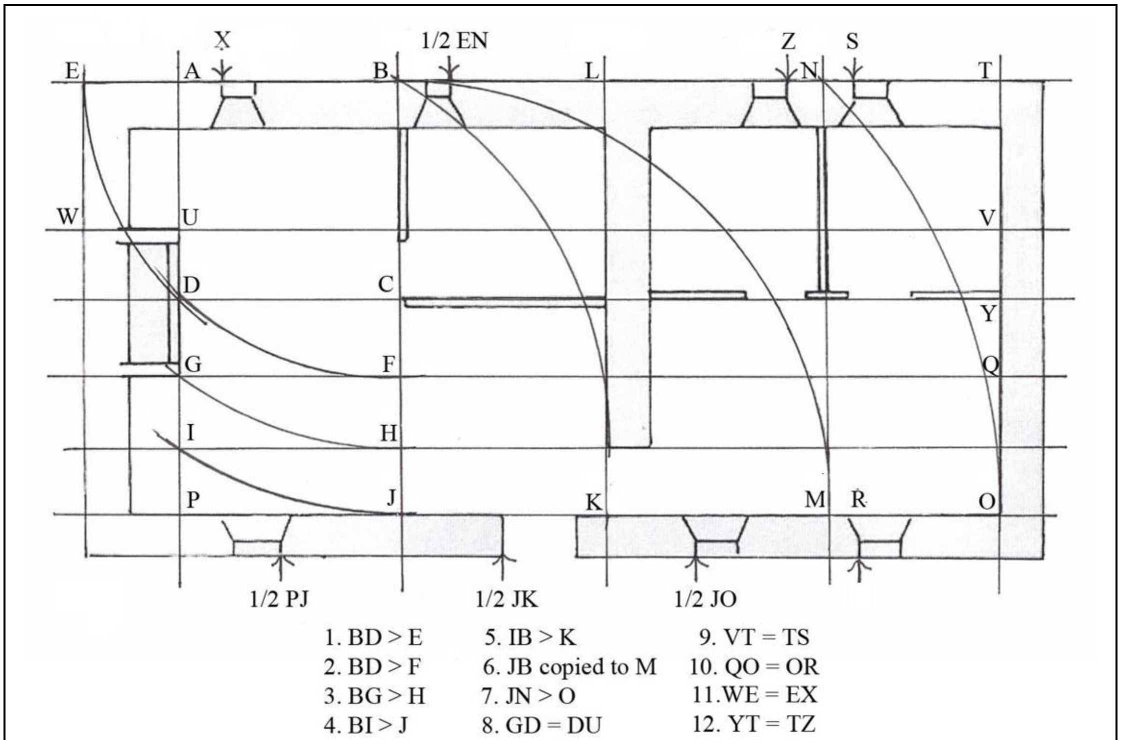


Fig. 85. Clooney Townland, North Co., Londonderry, ca. 1780.

original square and BD to F marks the hearth edge at G on the left (AD) side of the initial square. BG rotated to H marks a passage through the wall at KO that reminds one of the passage-way through the stone fire wall in the Henry Antes house that is marked out in exactly the same way. A final transverse step rotates BI to J to mark the front wall line.

Diagonal IB to K marks the transverse wall at J. Side DC marks the location of a line of lateral partitions across the right side of the structure. BJ is copied around node J to mark a square, BNMJ. Side NM of this square marks a partition along line NM. The pivot point advances from I to J and diagonal JN is rotated down to O to mark the right gable wall line.

Of the Irish trilateral rectilinear houses examined for this study, Clooney is the only one having its hearth on a gable wall. The window at S is marked by pivoting VT at corner T to TS and the window at R by pivoting interval QO at corner O to OR. The window at P is at one half of interval PJ and the window on the front wall between K and

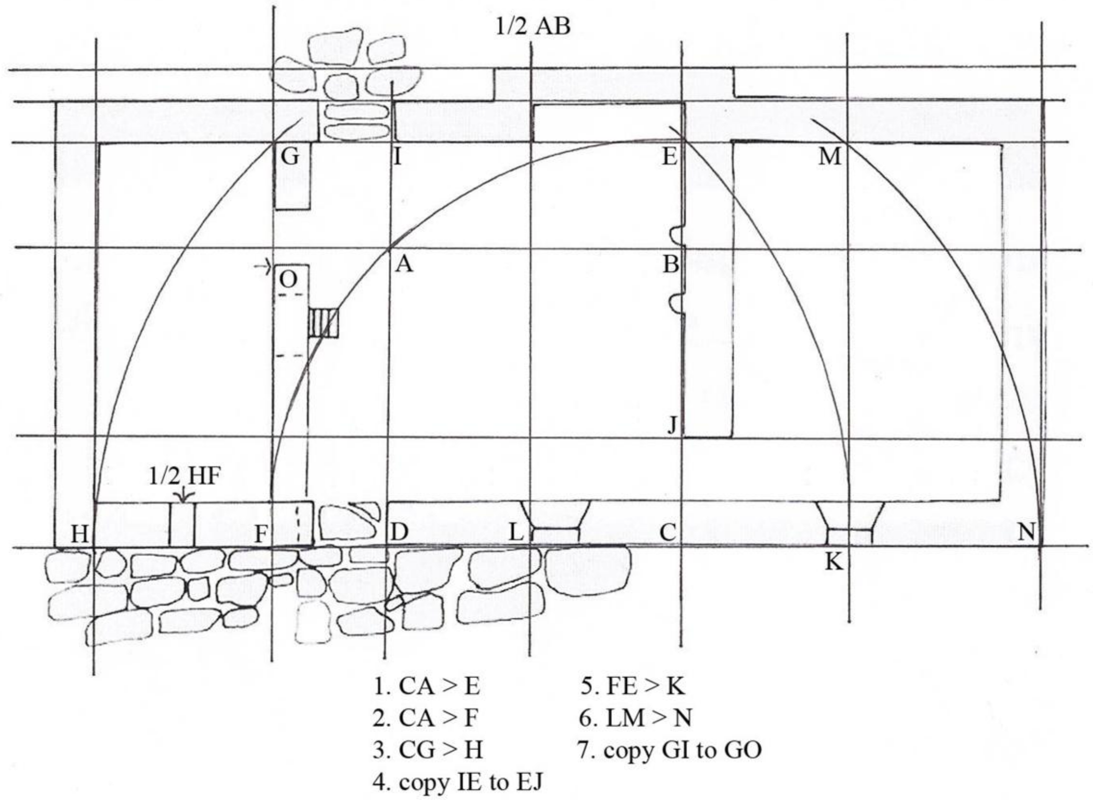


Fig. 86. Lunniagh Townland, County Donegal, Ireland,

M is at one half of interval JO. The front door is at one half of JK. Pivoting WE to EX marks the window at X remaining and pivoting YT at T marks the window at Z.

The house at Lunniagh in figure eighty six follows the same general format in a somewhat simpler way. The initial square is centered on the front wall line HN with the fireplace located at corner B. AC rotated to E establishes the width of the house by one step on the transverse axis and two steps to either side on the lateral axis establish the length of the house. On the left side line GF marks a full partitioning wall and on the right side line EC marks a partial partitioning wall. A through passage with front and rear door is marked by the AD side of the original square as part of line ID. IE is copied to EJ to mark the end of the masonry wall backing the fireplace thus allowing for passage to the room on the right. The outshoot on the rear wall and the front window at L are marked by a line at ½ of side AB of the original square. There is a narrow passage through the front wall at ½ HF on the left side and a window is centered on the far right side by the line at K.

It is in the Welsh long house that this more centered trilateral rectilinear pattern is seen at its most expansive. Two Welsh houses described by Iorworth C. Peate in *The Welsh House* as 'long houses' are significantly longer than the Irish houses and

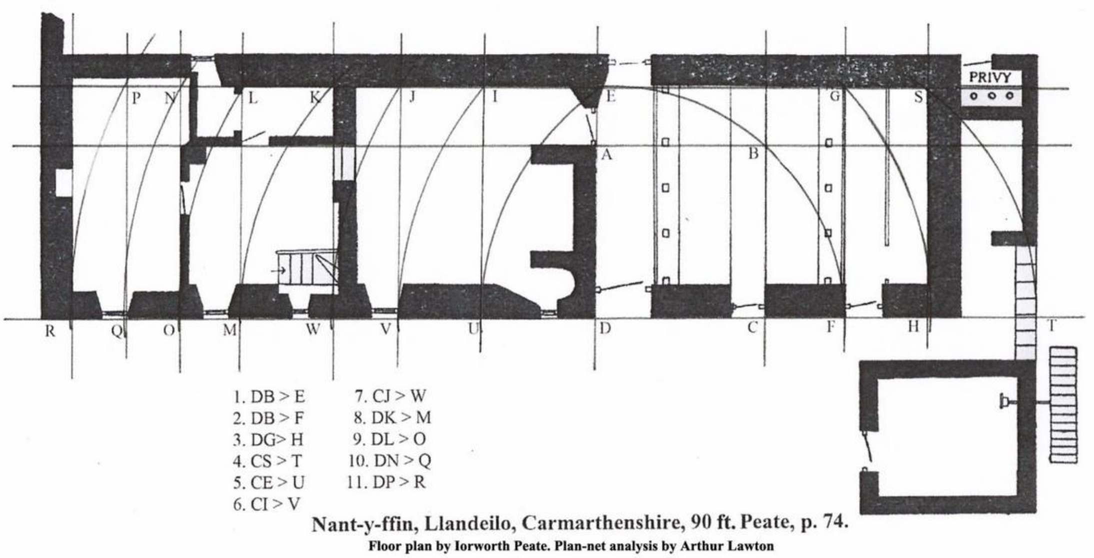


Fig. 87 Nant-y-ffin, Llandeilo, Carmarthenshire, Wales.

consequently have more lateral steps. In this Welsh house at Nant-y-ffin in figure eighty seven the fireplace backs onto the AD side of the original square. A single transverse step

DB rotated to E marks the gable width of the house, one lateral wall outside the line and one wall inside. The through passage is also marked by the AD side of the initial square. The first three steps pivot at node D as diagonal DB to E, DB to F and DG rotated to H. The pivot point then advances to C as CS rotates to T marking the right gable wall line of the house. The extension is substantially greater on the left side. From this same point C, diagonal CE rotates down to U, CI rotates down to V, CJ down to W and the pivot point then advances to D, rotating DL to O, DN to Q and DP to R marking the left gable wall line of the house. To accommodate the extreme length of the plan, the pivot point is moved a number of times, beginning at D and then to C both advancing to the right, then the direction from C is reversed to advance to the left, and finally the pivot point is advanced leftward to D to expand further to the left.

A Welsh long house at Ystradamen, Betws, Carmarthenshire in Wales, illustrated in figure eighty eight and described by Peate has a very similar floor plan with two outshoots on the long back wall of the house and an extension on the right gable whose purpose is not described by him. The initial square is located toward but not at the center and contain the fireplace inside the square on the BC side. This square is expanded by rotating diagonal CA in two directions to E and to F forming square LECF. This is expanded by rotating diagonal CG to H and then down to I. JI is then copied laterally to K to mark the left gable corner of the house. Diagonal CL is rotated on the left to advance on the transverse axis to M marking the extension of the two outshoots on the back wall. The rear wall of the left outshoot is tilted but the intent was probably to follow the lateral line at M. The back wall of the right outshoot on line M is true to the rest of the building. The lateral line at H marks a thickening of the rear wall of the house to accommodate the

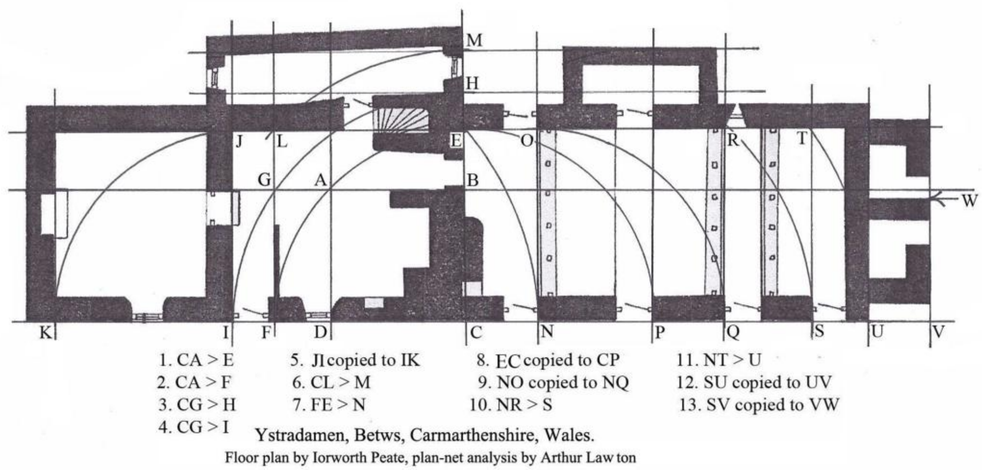


Fig. 88. Ystradamen, Betws, Carmarthenshire, Wales.

staircase. The right side of the building rotates diagonal FE of the first square expansion down to N to mark the right side of the through passageway. EC is copied to CP marking the front door and the rear door into the right side outshoot. The pivot point is advanced from C to N to copy ON down to NQ marking what appears to be a passage between cattle stanchions. Diagonal NR rotated down to S marks a door at S and NT down to U marks the right gable of the house proper. SU copied to UV marks the right side of the smaller structure appended to the main building.

This house is considerably more complicated, but it follows essentially the same plan-net process as does the house at Nant-y-ffin. There are eight steps of diagonal rotation, four steps that copy transverse intervals to lateral positions, and the pivot point is advanced laterally in one case from C to N. There are two steps of transverse diagonal rotation, CG to H and CL to M.

We have seen that plans with a high squareness coefficient tend to result in a quadrilateral rectilinear analysis. This is also true of trilateral rectilinear plans where the initial square base figure is located in either the right or the left corner. The plans with much lower squareness coefficient are of either bilateral rectilinear plans or trilateral rectilinear plans both showing the initial base figure not at the corners, but more to the center. They rely on copying and pivot point advance to gain the length expected in the tradition.

Fig. 89. Haus Hulscher, 33 Bucherstrasse, Thon. Photo by Rudolf Helm.

The final analysis in this chapter examines the use of plan-net analysis to resolve a practical issue in the interpretation of the architectural fabric of the house. The floor plan is of the Hulscher house at Bucherstrasse 33 in Thon, a village that is now a suburb of Nürnberg and was drawn by Rudolf helm. The house is long since gone now but Helm’s floor plan gives the room division as it existed in the late 1930s when he made the drawing and took a number of photographs. These houses with their roof structures supported by a system called *stehende Dachstuhl,* that is, a "standing roof truss system" are representatives of a type of structure common to house and stable and that were disappearing through the eighteenth and nineteenth centuries.

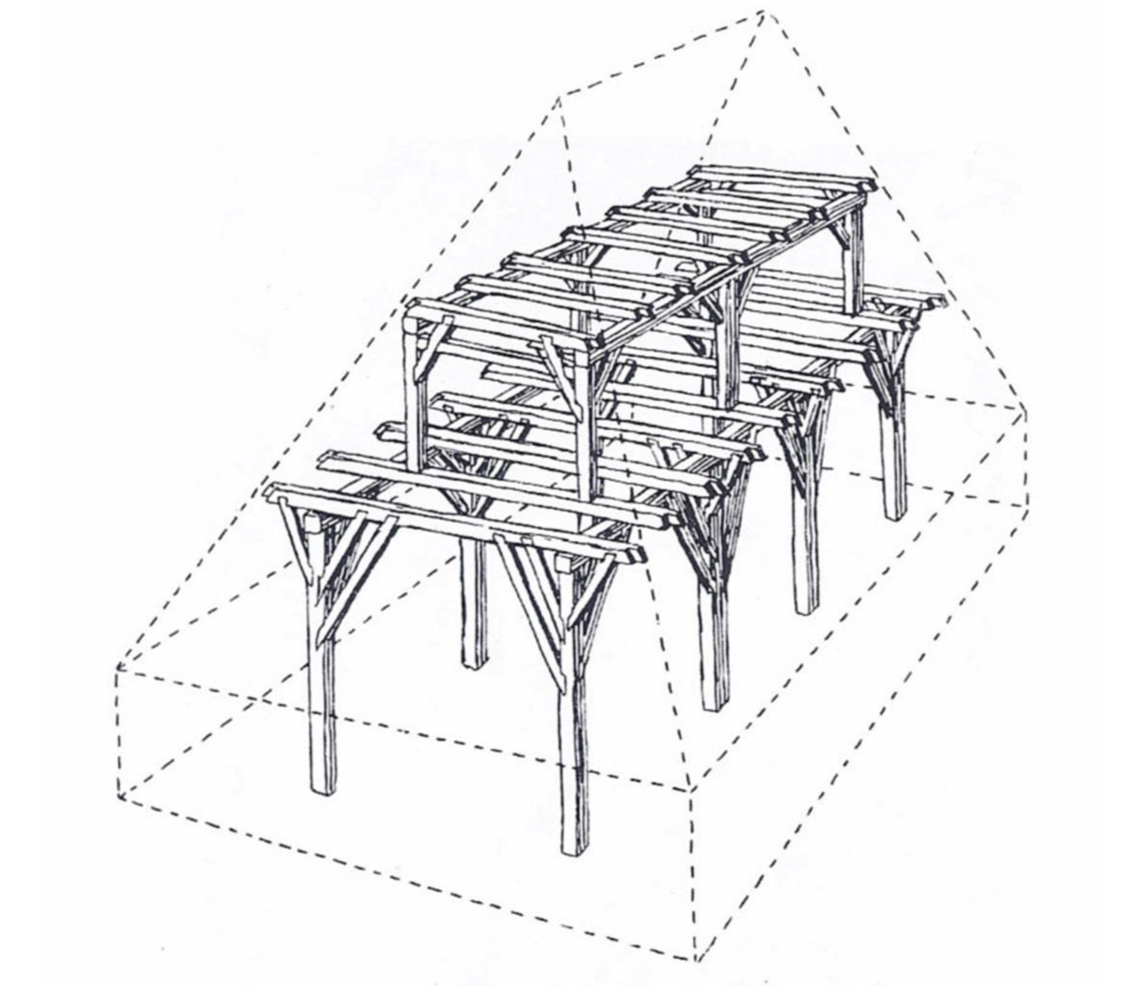


Fig. 90. *Stehende Dachstuhl*, standing roof truss structure. Drawing by Rudolf Helm

Helm notes that this form of house and stable are the same structure put to different purposes. Figure ninety is a drawing by Helm from his 1940 publication *Das bauernhaus im Gebiet der freien Reichstadt Nürnberg* that shows the importance of the massive standing posts undergirding this roof support system and that had an impact on the organization of internal room spaces. The massive roof covering, often called a mantel roof is supported on the posts and rests only very slightly on the very low outer walls. The area in the corridor down the center between the two post rows is called the *Tenne,* in English the threshing floor, but in a German house of the period house this area served as a main central hall or through passage, often dividing the animal stabling area from the human habitation area. Helm says about this house

> To judge from proportions, which we will consider next the house dates to the first half of the fifteenth century. At that time divisions of space still found today were generally customary, especially in the very remote areas. Therefore at least the two cross walls which separate the threshing floor from the dwelling part and the stall part are original and likewise the dividing wall between the kitchen and the parlor. The other walls are partially new and partially of undetermined age.[92]

Helm expected an entrance to the house at the narrower gable end, leading into the *Tenne,* the central hall that in the manner of the north German Hallenhaus ran directly under the roof ridge and between the two rows of posts in the center on the long axis of the plan. To either side of that would be the stalls on one side and the long dwelling space on the other side. This expectation is based on the idea that the Great Hall was open to the roof and that it was crucial to place the fire location at the place where the height to the thatched roofing was as great as possible. As may be seen in the floor plan in figure eighty three, the entrance is on the gutter side and the *Tenne,* whose space might be devoted to threshing floor, the central hallway or in the older form of house, the Great Hall, runs transversely across the two post rows and the ridge line between them. The fire location is in the expected place precisely under the roof ridge centered between the two post rows running parallel to the ridge. Helm argued with good cause that the two walls

[92] Rudolf Helm. *Das Bauernhaus im Gebiet der freien Reichstadt Nürnberg,* Berlin: Herbert Stubenrauch Verlagsbuchhandlung, 1940. 50, 51.

to either side of the *Tenne* were original and that the wall dividing the kitchen from the parlor, located directly under the ridge line, and with the hearth in the kitchen and the oven in the parlor was also original. The kitchen/parlor wall is an adaptation to the *Zweifeuerhaus* (two-fire house) system whose development followed the open floor-level hearth in the great hall and so would be an adaptation of the early fifteenth century

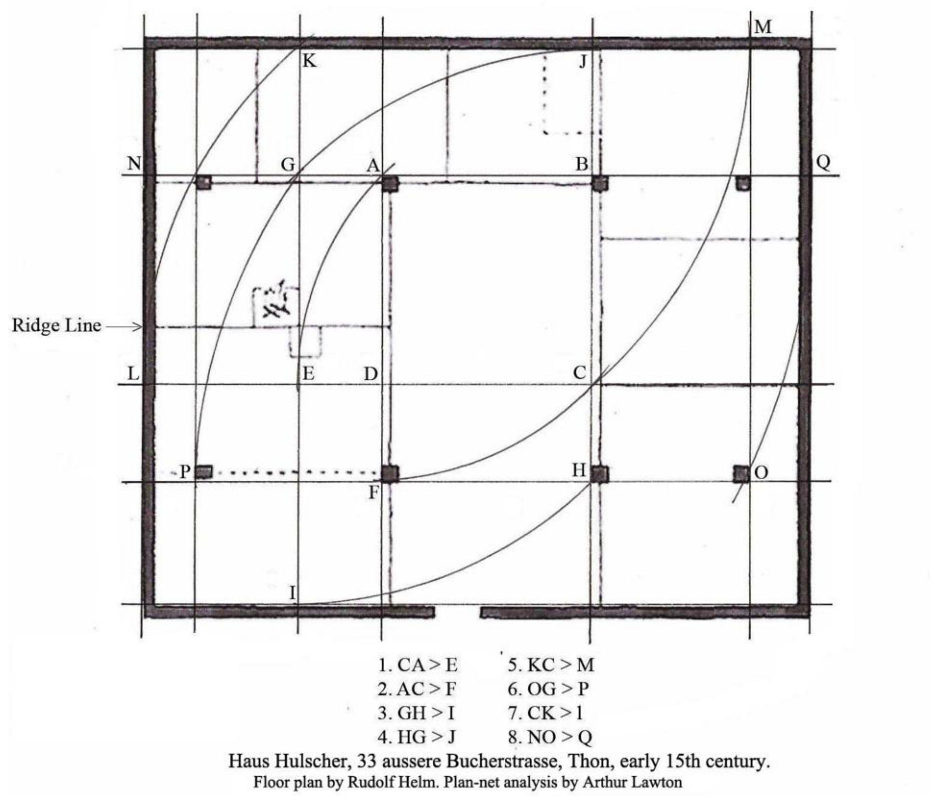


Fig. 91. Plan-net analysis, Haus Hulscher, aussere Bucherstrasse 33, Thon.

consistent with the construction of the house. As quoted above he said the other walls were of new or undetermined date. We will use plan-net analysis to determine which of the remaining walls were part of a unified whole that conforms as well to the post location pattern and the walls Helm deemed to be original and thus fits with the early fifteenth century period.

In figure ninety one the initial square ABCD sets the position of the two center posts of one post line at points A and B. The interval between A and B defines the initial square size and the diagonal of this square as CA is rotated to E to mark an edge line for the kitchen hearth, and as AC is rotated down to F to mark the other line of posts. This study has stated many times over that the initial square to some degree marks the location of the most significant cultural element, in the case of most houses, this being the hearth. This is not quite the case here. In discussion of the *stehende Dachstuhl*, Helm says the following;

> As corner supports of the truss work the outer posts must truly be especially strong. However, they are not so and to the contrary the diagonal bracing of these posts is less precise and in some cases is entirely missing. That means that in essence, only the inner square of posts can be considered the core truss work.[93]

Perhaps in the mind of the builder of Bucherstrasse 33, development of the system of posts was the most significant element. At any rate upon setting out the four corners of the initial square base figure he moved immediately in the next two steps to mark the location of the hearth and of the other post row at E and F. The balance of the plan development moves ahead from this point in a logically connected sequence of developments. GH to I marks the front wall and in the reverse direction, HG to J marks the rear wall. KC to M marks the outer two posts on the right side and OG to P marks the outer two posts on the left side. CK to L marks the left gable wall and NO to Q marks the right gable wall. It should be noted that while it is quite similar, the post to gable wall distance from the post at P is not the same as the post to wall distance from the post at O.

[93] Helm. p. 49.

Also the inside measurement from post P to Post F is not the same as from post F to post H, and from post H to post O is shorter than the other two. Yet all these variances are can

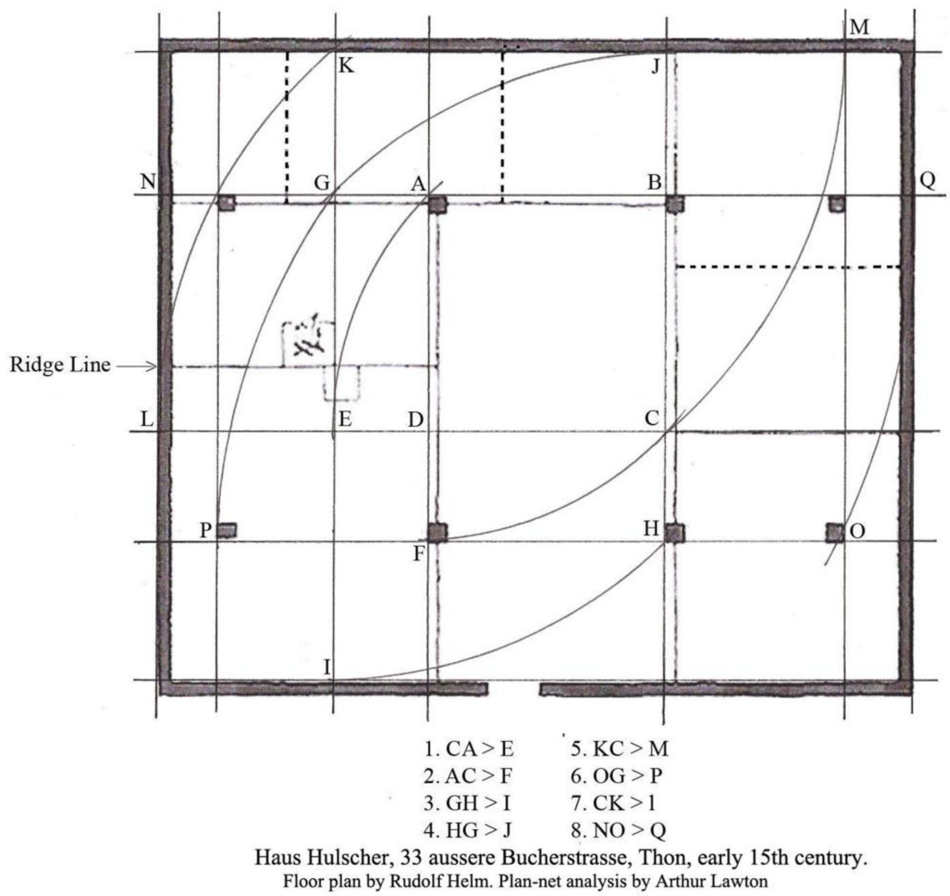


Fig. 92. Dotted lines show partitions in the Helm plan that do not conform to plan-net lines. Hans Hulscher house, aussere Bucherstrasse 33,Thon.

be explained as being in conformity to plan-net lines.

The stage is now set to solve the problem in interpreting the architectural fabric mentioned above. Figure ninety two shows the room division as Helm recorded it in the 1930s, the dashed lines indicating the walls that do not coincide with plan-net lines; a vertical line between N and G, one between A and B and a horizontal line between B and C. They do not match plan-net lines implying that they are likely to be later modifications to the floor plan. The continuation of the HCB wall between B and J is open to question, as it coincides with a plan-net line but the wall on the other side of the *Tenne* does not continue in the same manner. The space on both sides of this line was very likely devoted

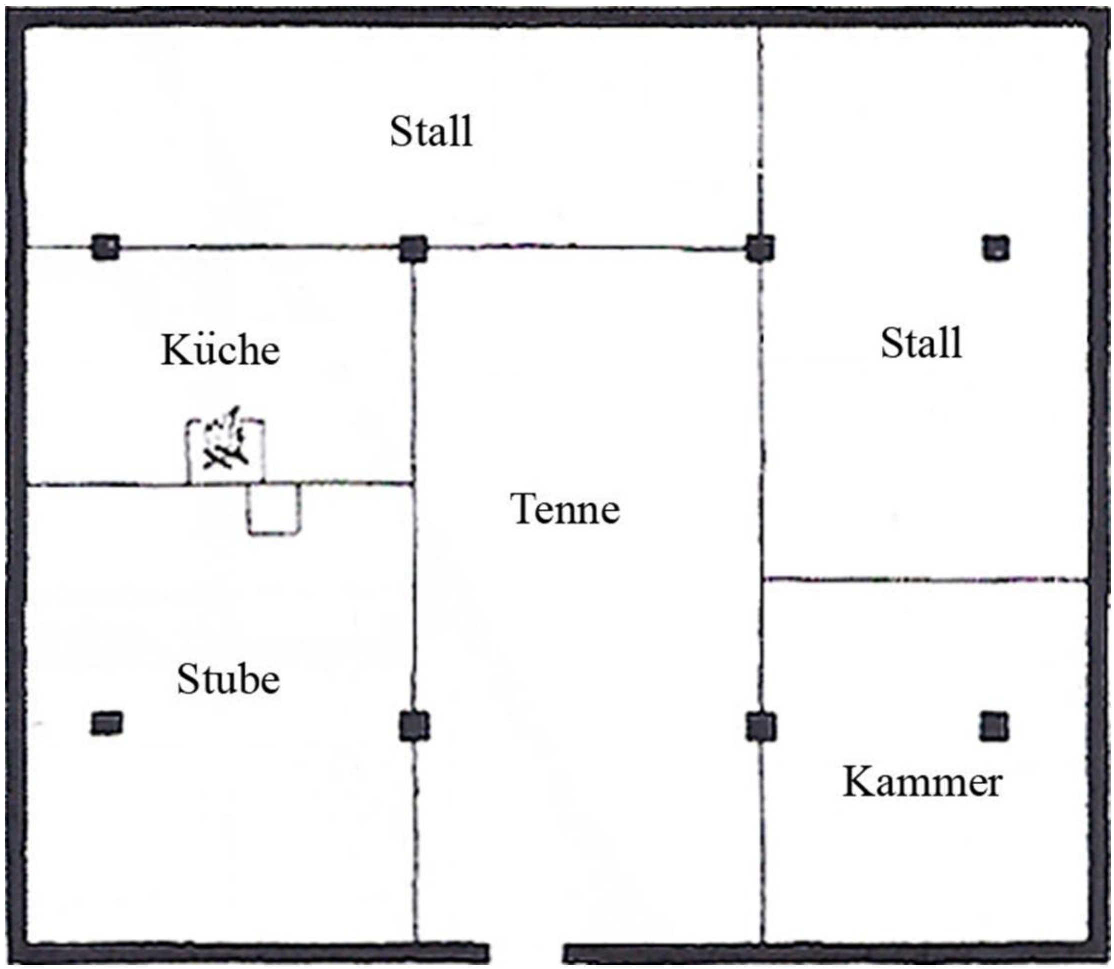


Fig. 93. Reconstructed floor plan based on plan-net analysis, showing partitions that align with plan-net lines. Hans Hulscher house, aussere Bucherstrasse 33, Thon.

to stall space and consequently could have been open without a wall or separated by a wall to provide stall space for different animals. These changes to the floor plan bring it into conformity with most of the other floor plans drawn by Helm in this study. Figure ninety three presents this floor plan as reconstructed based on plan-net lines and indicates the most likely use of space traditionally associated with this floor plan. The

Einfeuerhaus (one-fire house) was a house with a floor level open hearth that protects the thatch roof by locating the fire under the ridge line and placing a *Rauchfang* (smoke and spark shield) between it and the thatch. The transition to *Zweifeuerhaus* left the fire location in the same place and separated the open fire from the smoke-free parlor by the expedient of a wall between hearth and oven with a *Feuerloch* (fire hole) through the wall. The two side walls of the *Tenne* separate the original three functions of the Germanic house, human dwelling space, threshing floor for processing grain crops and stall space for the husbanding of animal in a cold climate in which animal heat contributed to warming the whole. These three walls are thus established as consistent with the building construction of the early fifteenth century. The three partitions that were eliminated in the reconstruction drawing open up the stall space in a practical way to accommodate large animals, leaving one wall that may or may not have been original, a matter that necessarily must be determined by reading the architectural fabric, since coinciding with a plan-net line does not guarantee that it was original. The other two wall lines, HB and the horizontal line at C separate dwelling space from stall space.

Confirmation of this reconstructed floor plan is found in a *Bauzeichnung*, a drawn building plan photographed by Rudolf Helm and archived among his papers at the *Deutsches Nationalmuseum* in Nürnberg. The plan was required by the Nürnberg *Forstamt* in 1773 for timber to construct this building at Nennhoff. Note in figure 94 that the distribution and function of rooms is exactly the same as in the reconstructed plan.

The room use in both cases takes the following form;

Stall

Küche T Cammer
e
n
Stube n Cammer
e

32 Schuh lang

29 Schuh breit

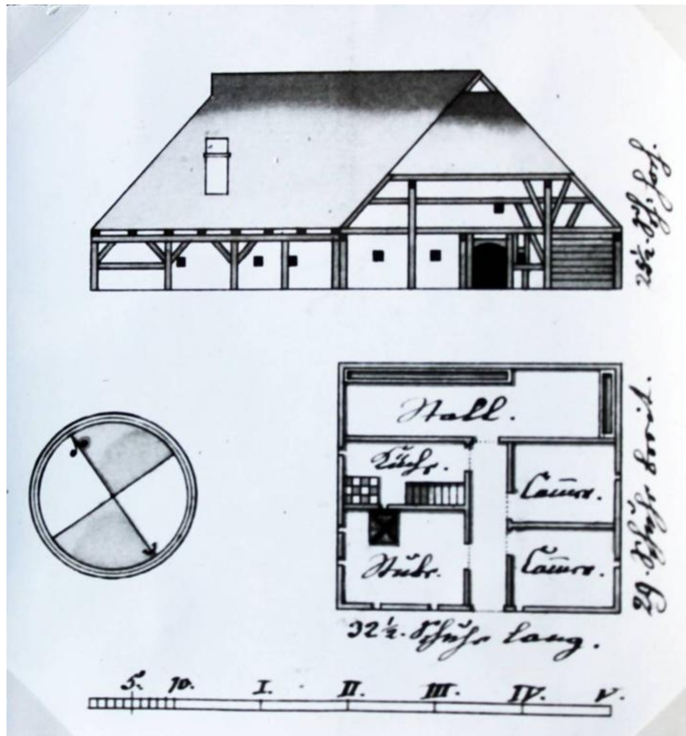


fig. 96. The Nennhoff floor plan is identical to the Thon floor plan.

Stall is the stabling area, *Küche* is the kitchen, *Cammer* means room, *Stube* is the parlor and *Tenne* refers to the threshing area though use of this space is not limited to threshing in all cases. The building is 32 feet long and 29 feet wide.

Plan-net analysis can be used to graphically demonstrate and compare the implicit messages of architectural form. We return for a moment to the 1798 mathematical formula of Lorentz Suckow applied to a baroque building designed by David Gilly in Figure 94a, and to the Villa La Rotunda of Palladio in Figure 94b. The two buildings are symmetrical, but differently so. Linear symmetry occurs across a building facade when there is a central element and similar or identical elements carefully balanced to both sides of the center point. Centralized symmetry occurs when all elements are equally balanced as they surround a center point. In buildings that are not round, this occurs most readily around a square, a hexagon or an octagon. Both communicate an implicit message about power, social position and, as plan-net analysis indicates each implies a different view of the relationship between hierarchical levels in the conceptual

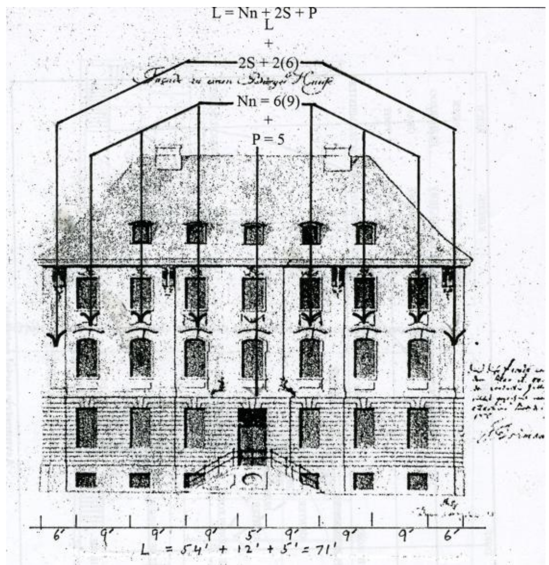


fig. 95a. When linear symmetry is diagrammed it communicates power that distributes from the top down.

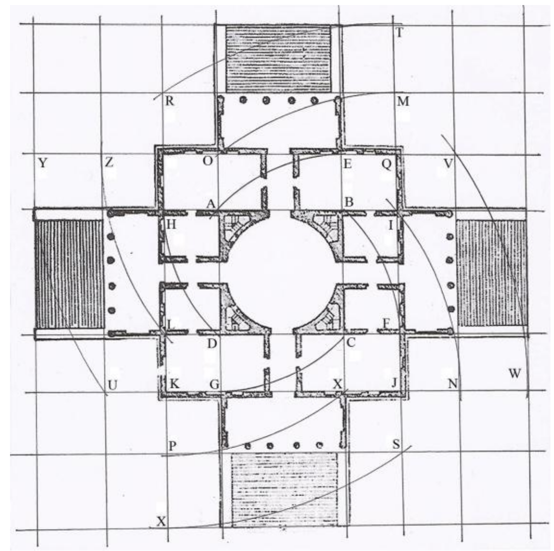


fig. 95b. When centered symmetry is diagrammed it communicates power that distributes from the center outward.

organization of space. In Suckow’s case, the entire analysis is reducible to a common module that can be divided into every element in the analysis. In his equation the term L contains every individual element of the formula as potential, each to be distributed out appropriate to its position in the hierarchical structure. The impact of this symmetry occurs upon externally viewing it, as in the facade.

In the Palladio structure, though at first glance one might think it is a grid of equal squares it is not so. The length and width of the quadrilateral figures in each rank of the hierarchy outward from the center are the result of a different geometrical construction and so shape varies slightly from rank to rank. However, all the small differences are equally balanced by their distribution to the four cardinal directions. One becomes fully aware of the symmetry only from the interior, increasingly so as one approaches the center.

It is not our purpose to fully explore the ramifications of symmetry but here, rather to show how a plan-net analysis can be the basis for such exploration. Having looked at some of the ways in which such analysis is useful, we close by putting this methodology into a larger context. Where does this methodology, so useful for vernacular design and construction techniques that were restricted by limited calculating skills and drafting technology, fit into the larger context of pre-modern building construction? A reasonable view is that the vernacular designer/builder used from the same tool box as did the master designer and builder of cathedrals and monumental architecture, but that he selected only those tools that solved the problems that he faced in his world of vernacular buildings.

This possibility becomes clear in a careful look again at Robert Bork's analysis of the Strasbourg Cathedral facade of 1250-1350. We find here plan-net techniques identical to those described for vernacular buildings. His analysis shows a root rectangle BGFC constructed by rotating the diagonal of square BENC. I have added line EN and the letter N. Rectangle BGFC is then expanded by rotating its diagonal to H creating a $1 : \sqrt{3}$ rectangle. This kind of expansion is commonly found in plan-net analyses and these observations signal that Bork is using plan-net methods though he has not presented the concept of a network of parallel, rectilinear lines generated from an original square base figure. Considering just the section of the facade Bork shows us here, we will designate square ABCD as the initial square base figure. Square BENC is copied from ABCD by rotating AB and DC around B and C to BE and CN. Diagonal CE is rotated to F forming a 1:√3 rectangle and then diagonal CG is rotated to H creating 1:√4, that is, 2, doubling the interval DC. Thus the cross-section at ground level has been laid out as a sequentially proportional extension by copying and by diagonal rotation from square ABCD. On the vertical axis and confirming the numerical intervals noted on the left side of Bork's analysis, diagonal rotations from corner D follow as DB to I, DH to O, DJ to K and DL to M, creating the same kind of relationships interconnected by sequential proportionality.

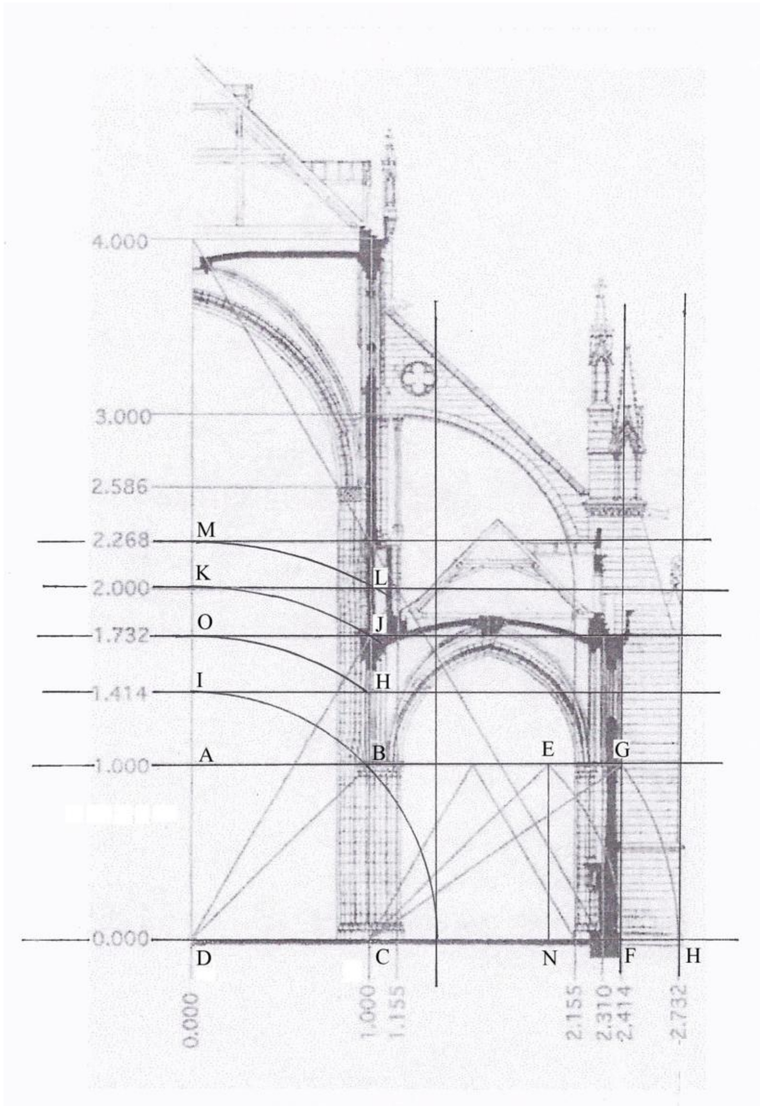


fig. 96. Robert Bork's analysis of Strasbourg Cathedral, facade considered as a plan-net analysis.

Because the network of lines is generated in two directions and on two perpendicular axes, this is a bilateral rectilinear analysis generated from the left hand corner, though analysis of the entire facade might result in a trilateral analysis. As a bilateral rectilinear analysis, it is no different than the analyses presented earlier for Weissenbrunn and Haimendorf and it is reasonable to think that the vernacular designer/builders at Weissenbrunn and Haimendorf were using the same geometric tools as were the master builders at Strasbourg some two or three hundred years earlier.

The cathedral is vastly larger and more complex. Its plan-net characteristics constitute a broad proportioning schema within which many other geometric design tools were applied. Vernacular craftsmen chose and used design and construction control tools that they understood and knew to be useful from the same tool box as the master builders working on Strasbourg Cathedral. The men that produced Weissenbrunn, Thon and Haimendorf were masters who thought a building, not in terms of recorded numbers, but who thought it in their own way, as masters of the manipulation of geometric figures.

In this chapter we have examined twenty four out of a total of some fifty analyzed plans, demonstrating the remarkably close correlation between the lines of the selected floor plan and those of plan-net analysis whose geometry is based on manipulation of the root rectangle. By no means are all plans subject to successful analysis in this way. Classification based on vectors of plan development radiating from the initial square base figure results in three forms of rectilinear analyses but there were no examples found of the two possible forms of linear plans. To the extent one accepts that a successful analysis demonstrates the designer/builder's plan development method, plan-net analysis has value in interpreting problems in the fabric of the existing building. Analysis can also indicate the implicit meaning of architectural forms especially in the use of or the lack of symmetry. Finally, the methods of plan-net analysis are best understood as a part of the larger context of architectural planning and construction, being those elements that were of use to a vernacular builder because they did not depend on arithmetic calculation.

CHAPTER SIX

Conclusions

This dissertation proposes a geometrical design method for vernacular floor plans that accounts for the pattern of certain vernacular floor plans. The distant origin of the method can only be hypothesized, but descriptive textual and iconographic evidence suggests its likely form and the extent to which it was transmitted through time. Over time it was eventually symbolized in the ritual and paraphernalia of the modern Freemasonry movement.

Until very recently, academic attention to geometry in design and layout of pre-modern buildings focused on a search for the cause of harmony and beauty in buildings. Many such studies sought to find favored geometric figures incorporated in the lines of the structure. Theory was tested by superimposing the favored figure on the floor plan and looking for coincidence of significant points of the geometrical figure with significant elements of the plan. Building on new directions undertaken since the middle of the twentieth century this dissertation examines instead how the proposed methodology was used by the Pre-Modern designer/builder to design plans, lay-out ground lines and convey information in the construction process. This approach tests its theory by demonstrating that the proposed analytical method successfully constructs an accurate copy of the floor plan pattern under analysis.

The available documentary evidence is insufficient to prove categorically that the plan-net method was used create specific floor plan or lay out a particular set of ground lines. On the other hand it is evident in the twenty four floor plans analyzed for this study that the method can reproduce the pattern of the floor plan accurately and can account for placement of details such as windows and doors. Thus the documentary evidence at this point is slight but there is strong circumstantial evidence provided by the analyses that plan-net geometry correlates closely to the results of vernacular floor plan design and lay-out practice. If there is no practical connection between plan-net geometry and the design process for floor plans, it does beg the question of what accounts for the strict geometrical conformity of plan-net lines displayed in the analytical drawings to the actual floor plan under analysis.

Summarizing the method very briefly, all plan-net analyses originate from a relatively small square figure either by rotation of the diagonal of quadrilateral figures or by copying previously generated intervals to adjacent locations by rotating the interval around an end point. Such design steps taken by divider and comparable straightedge and cord and pegs steps taken to lay out ground lines prior to construction are exactly the same. Only the scale differs and this identity of steps eliminates the need for calculation when scaling up from design to construction. Plan-net steps produce a network of rectilinear lines whose intersection nodes either make available the means for subsequent generational steps or locate new elements in the plan.

Floor-plan analyses can be differentiated either as linear or rectilinear since a plan-net consists of a growing network of rectilinear lines advancing from an original square figure, A linear plan develops out of the original square base figure in one or in both directions along a single axis only. A rectilinear plan develops out of the base figure along perpendicular x and y axes in one or both directions on both axes. Because all analyses begin with a square there are four directions in which a plan can develop. Linear and rectilinear plans can be either symmetrical or asymmetrical, though symmetry may well be approximated visually but not necessarily created perfectly so.

Individual plan-net steps are either generative or locational. Generative steps add new space to the net by rotation of the diagonals of a quadrilateral, by copying an interval already constructed or by advancing the rotational pivot point. Rotating the two diagonals of an established quadrilateral figure adds new space to the net by pivoting the two diagonals to a position overlaying the two sides, thus extending the quadrilateral in a square root relationship. Copying an interval is always to an adjacent location since it is done by rotating the interval's line segment by 90^o or by 180^o around a line segment end point. Pivot point advancement generally occurs along the lateral axis in connection with rotation of the diagonals. Lines of the plan-net resulting from these generational steps determine the position of the perimeter walls and usually the inner partitioning divisions. Exterior walls, interior partitions and fireplace jambs may occur inside or outside the lines but are not considered as centered on the lines because builders generally built to a line. Placement of pillars or posts might be an exception to this though as was seen in chapter five regarding the posts of the Hulscher house in Thon, these posts appear facing but not centered on the line.

Windows, doors, hearths and other architectural elements are located along relevant plan-net lines. Their position, if not determined by intersection nodes formed by intersecting lines, is determined by locational steps. Locational steps copy an interval already defined on the plan-net to an adjacent location along a line by rotating the interval around one of its end points. Width and height of windows, doors, staircases and the like can just as well be determined by modular measurement from one side or the other of the locational point determined by a plan-net step. Locational steps occur in two ways, either by rotating an existing interval 180^{o} around one of its end points to create a new point on the same line or by rotating the interval by 90^{o} around a corner to create a new point on a perpendicular line. Copying expresses a relationship of 1 : 1. Locational steps do not add new space. They divide space that is already marked out.

Generative plan-net steps together with locational steps constitute a unified system that unfolds in a logically connected sequence. Each added space is generated from elements previously laid out in the plan-net, suggesting a meaningful correlation between a developing plan-net and the developing leaves and stems on a branch for which the adjective organic is appropriate. Because each subsequent step is governed by information presented in the previous step it is hypothetically possible to reconstruct the method by which the plan was designed provided the initial base figure is correctly identified. To follow a sequence of generative steps from the initial square to the completed and accurate reproduction of the plan validates the analysis. Locational steps on the other hand are not sequential except in the sense that they follow the establishment of lines and nodes marking the interval they are copying or dividing. When the locating point for an architectural element is a simple half or quarter of the interval between existing intersection nodes it is reasonable to think that element was located according to the plan-net.

The continuously increasing number of lines and nodes as the plan-net develops provides increasing opportunity for the designer to choose among options. These options include selection of diagonals for rotation and selection of intervals either for copying or for dividing. Availability of these options provides a simple and practical methodology that allows many options to meet individual needs and desires. The increasing cascade of available nodes provides more and more opportunity to vary the dimensions and details in the plan. The degree of flexibility available seems to allow a common methodology to serve the specific expectations of a wide variety of local cultures and chronological periods.

When sophisticated calculating skills and large measured drawings were lacking, geometrical methods solved architectural problems and transmitted architectural information at the building site. Since these methods were taught by hands-on practice in a master/apprentice context rather than by textual learning, written texts did not exist until printed sources began to appear in the late fifteenth century. The limited documentary evidence available suggests the plan-net as a hypothesis that could be tested by applying the plan-net to data that consisted of published floor plans. Floor plans reconstructed this way were repeatedly in accord with the observed data.

Iconographic evidence for the method was traced from its origin as a practical application in the ancient world to its final stage as symbolic remembrance in the Freemasonry movement of the Early Modern period. Building on references in Egyptian foundation texts to "casting the plan-net," the present study developed a hypothetical method for "casting the plan net," as an alternative way to analyze vernacular floor plans. This analytical direction was furthered in so much as by representing J. Marshall Jenkin's square root methodology as a rectilinear network, that is, a plan-net, this served to unify his aggregate analytical steps into a unified systematic process. The 'Dream of Gunzo' miniature illustrates this rectilinear network methodology at work. The Weissenau Codex demonstrates transitional phase symbolizing cords as the means of transmission from divinity’s ideal plan to the mortally practical plan. The symbols on the apron of the Order of Freemasonry and the title page of the Order of the First Masonic Degree present the tools and methods of plan-net geometry as symbolic remembrances of a transmitted heritage of architectural methodology, bringing to completion the transition from practical process to symbolic remembrance.

The application of geometry to architecture has been a subject of academic discussion for centuries, but until very recently it was directed to the search for ideal beauty in architectural forms. This study proposed that geometric methods in architecture were addressed by the construction trades to the solution of practical architectural problems and by scholars and theoreticians to the expression of ideal beauty. Applicable academic literature was assembled identifying the thread of developing geometrical understanding that contributed to plan-net practice. Alexander Badawy’s translated and discussed Egyptian foundation texts describing the plan-net method. Tons Brunés assembled developmental steps of geometrical practice relating to practical manipulation of the square and its elements. The Vedic Indian Sulva Sutras document cord and peg methods that manipulate geometrical shapes to solve architectural problems. Richard Tobin’s hypothetical reconstruction of the Canon of Polykleitos demonstrated proportionally sequential manipulation of geometrical shape serving to extract the proportions of the sculpted human body from an initial square base figure. The sketchbook of Villard de Honnecourt documented finished figures conceptualized as assemblage of lines governing the final form. Finally Lon Shelby’s examines how Mattheus Roriczer’s geometrically extracted the finished form of a pinnacle from a square base figure. Just as Roriczer extracted the completed figure of the pinnacle from the square base figure, so plan-net geometry extracts the completed floor plan from the initial square base figure.

Plan-net analysis was applied to twenty four floor plans from widely varying Pre-Modern cultures and time periods. An initial square base figure was identified consistent with the relevant lines of the floor plan. From this initial figure a plan-net was extracted, by which was marked a completed floor plan, in each case essentially identical to the plan under analysis.

Plan-net analyses can be grouped according to the vectors representing the direction of plan development as it advances outward from the initial square base figure. Since all analyses begin with a four sided square, plan development can occur along some or all of the four vectors. Thus plans are called unilateral, bilateral, trilateral or quadrilateral. Development along only one axis is linear and along two perpendicular axes is rectilinear. No linear buildings were analyzed. For plan development on a single axis only, the initial square must comprise the full width of the building. The plans presented in this study are bilateral, trilateral and quadrilateral rectilinear plans.

Within a given class the broad laying-out lines are similar for buildings of the same type but they are not identical and generally the plans differ in their details. Similarity of the broad lines communicates design practice acceptable in the local community. Differences in the details reflect the individual needs of family, the dictates of economic and social standing, of agricultural and craft demands and so forth. Sharing a common plan-net methodology correlates with membership in common community. Both building and community stand as unified wholes, all parts and members of each being present in the "right measure." Differences in the details acknowledge that each building reflects individual needs and desires.

Plans selected for this study begin with two early eighteenth century Pennsylvania German houses, Bertolet and Antes, sharing similar floor plans but varying in plan development. These uncomplicated plans demonstrate diagonal rotation, interval copying and interval division. Variation in their use demonstrates how the same system met needs that differ across social boundaries. The Bertolet builder was an immigrant settler beginning a new life in a new world and Henry Antes was a successful immigrant landowner, carpenter and mill builder from Palatine *Bürger* family of substantial social standing. The following analysis of Andrea Palladio's fifteenth century Villa La Rotunda plan, restricts development to the step of rotating the diagonal of the square, marking out the lines of a very formal building that is completely symmetrical around a center point. This is a quadrilateral rectilinear analysis whose only difference from the Bertolet and Antes analyses is its limitation to diagonal rotation for every step, each vector treated in the same manner. Thus symmetry or asymmetry is a function of how the designer uses the plan-net options available.

Buildings within a single analytical class such as quadrilateral rectilinear, share similarities deeper than house type or chronological period. Nearly all these analyses place the most significant cultural element of the plan in a centered location, whether it is the hearth space or sacred or public space. The balance of the plan is organized around the centered element. This may be seen in quadrilateral rectilinear analyses of the Pennsylvania Herr house, the Nürnberger Schultheiss house, the Church Farm house in England or the undated German Wittenschlager stable. The centered element in the three houses is the hearth and in the stable it is the location of four core *stehende Dachstuhl* posts. Quadrilateral rectilinear analyses are associated with a high squareness coefficient so these plans are square or nearly so, to be expected of a house whose rooms are distributed around a centrally located hearth.

Seven buildings, the Eckstein, Sperber, Hupfer, Ottörfer, Buchner, and Heberlein houses, and the Steinbuhl distillery demonstrate bilateral rectilinear plans. These plans also show a high squareness coefficient. The analysis proceeds from a square base figure that forms a corner of the plan, developing along two perpendicular axes and each in a single direction only. The initial square base figure may be located at any corner of the plan. Helm, noted that these seven bilateral house plans are cruciform. The plan suggests two perpendicular axes in the arrangement of partitions that come together near the center of the plan. This visible cruciform structure in the floor plan implies the underlying perpendicular axes on which the plan is analyzed.

With the exception of the distillery the hearth location is associated with the initial square base figure, occurring on the perimeter of the base figure and not within the square. These buildings, all from the same culture area, are *Zweifeuerhäuser* (two fire houses), the hearth separated from the heating oven by a wall with a hole through which the oven is stoked from the hearth. The similar analyses of these buildings demonstrate a design methodology pervasive to this culture area and chronological period.

Bilateral rectilinear development produces plans with a high squareness coefficient if extensively developed along two perpendicular axes, but with a low coefficient if minimally developed on the transverse axis and extensively developed along the lateral axis. Late seventeenth and early eighteenth century Anglican Virginia

Churches published by Dell Upton are such buildings of a linear appearance. Analyzed here are Mangohick Church, Middle Church, St. Paul's Church and St. Luke's Church. All analyses begin with one or two transverse rotational steps to develop the full gable width of the building. All subsequent steps are in a lateral direction and develop the length of the building. At Mangohick all lateral steps are by rotation of the diagonal and in the other churches by a combination of diagonal rotations and interval copying. The initial square base figure ranges from one half to three quarters of the gable width and it accommodates the altar location, the significant cultural element of a church. Equidistant buttresses along the side walls of St. Luke's Church suggest they were carefully measured by ruler. However, the distance from the set of buttress to each corner differs by about the thickness of the gable wall. One could argue one end as measured from inside the gable wall and the other from outside the gable wall but the precise fit of all plan elements to one or the other side of plan-net lines strongly suggests plan-net geometry.

The four church analyses are bilateral rectilinear because the square is located in the corner. Advancement can occur in one direction only, minimal on the transverse axis and maximum on the lateral axis. In trilateral rectilinear plans the initial square base figure is more central at some point along the lateral axis. Lateral development occurs in both directions. Options for transverse development remain the same. These buildings also visually appear to be linear, developing to considerable length in Irish and Welsh rural dwellings as noted in Ellen Cutler's house, the house at Clooney and Lunniagh, and at the Welsh houses of Nant-y-ffin and Ystradamen.

All hearths but one are located in the interior of the plan, Clooney's hearth being on the gable. The hearths are positioned by some edge of the initial square base figure, though position relative to the square varies widely. Interval copying and pivot point advancement occur more frequently in these plans because to extensively advance an analysis in the same direction from a single pivot point results in a decreasing advance with each succeeding step, a case of diminishing return. These houses may copy an interval to advance laterally along the axis, usually taking the full transverse plan-net interval copied to the lateral line thus forming a square within the plan. A common alternative for lateral distance is to advance the pivot point along the lateral line in one or both directions. Trilateral rectilinear analyses advance laterally in both directions and transversely in one direction.

Plan- net analysis is helpful in determining original lines and features when a building has been altered or a part of it is missing because in successful analyses it can recreate the lines governing the design of a building. Analysis of the house at *aussere Bucherstrasse 33* in Thon is a case in point. Rudolf Helm photographed the house and drew its plan in the 1930s before it was demolished. Three partitions in the plan he labeled of indeterminate origin. The plan-net analysis accurately marks the location of the eight posts of the *stehende Dachstuhl* as well as the exterior walls. The hearth and heating oven with the partition between them are located as expected immediately under the ridge line. One hearth edge is located on the first extension of the initial square base figure that in this case locates the inner core posts of the roof truss system. The indeterminate partitions do not fall on plan-net lines, implying they were not a part of the original plan. The reconstructed floor plan removes these partitions, opening up the stall areas for the accommodation of large stock and resulting in a plan entirely consistent with period use of space.

Plan-net methods are based on a geometrical understanding of measurement, significantly different from modern modular understanding of measurement that records as numbers and sets marks all dimensions by an infinitely divisible and standardized unit of measurement. The great advantage of the number system is that its infinitely divisible numbers can model process or shape to any degree of precision. The requirement for advanced calculating skill and for the technical means to utilize large scale measured drawings on the building site was a barrier to its widespread use in vernacular building design and layout. Once the investment is made in calculating skills and literacy, the designer/builder has in hand tools that enable one to shape and enclose space precisely to whatever the inner creative vision might demand. Geometry on the other hand, touches on the hidden order of the cosmos, rendering up for use the parameters by which humans have always brought order to their activities by organizing their space.

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Badawy opens with a discussion of tradition and variation in ancient Egyptian building design that includes a presentation of the inscription evidence for the concept of the plan net. Badawy proposes a theory of harmonic design and discusses related geometrical problems. He then applies his theory of harmonic design to a number of Egyptian monumental buildings.

Baumgarten, Karl. *Das deutsche Bauernhaus. Berlin: Akademie-verlag, 1980.*

A historical-developmental approach to peasant houses that uses generalized line drawings of overall house forms to show variety within type. There are good floor plans and elevations with pen and ink drawings of houses on the landscape that are cited as to their photographic original source.

Bishir, Catherine W., Charlotte V. Brown, Carl R. Lounsbury and Ernest H. Woods III. *North Carolina: A History of the Practice of Building.* Chapel Hill: The university of North Carolina press, 1990.

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Dehio, Georg. *Untersuchungen über das gleichseitige Dreieck als Norm gotischer Proportionen.* Stuttgart: No publisher given. 1894.

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Elkar, Reinar S. and Gerhardt Fouquet. "Und Sie Bauten Einen Turm…, in *Handwerk und Sachkultur im Spätmittelalter.* Wien: Verlag der Österreichischen Akademie der Wissenschaften, 1988.

This essay provides detailed "observations on everyday material culture in a small German city in the Middle Ages," by means of its study of the processes of civic organization, material assembly, work crews, their schedules, wages and food, and the actual construction process of the tower for the Nikolaikirche in Nürnberg.

Ellis. Richard S. *Foundation Deposits in Ancient Mesopotamia.* New Haven: Yale University Press, 1968.

Fleischmann, Peter. "Arbeitsorganisation und Arbeitsweise im Nürnberger Bauhandwerk" in *Handwerk und Sachkultur im Spätmittelalter.* Wien: Verlag der Österreichischen Akademie der Wissenschaften, 1988.

In one of a series of articles, Fleischmann provides a detailed examination of the the procuring and assembly of materials and the process of construction of a modest craftsman's house on a very small building lot in Nürnberg in the sixteenth century.

Frankl, Paul. *The Art Bulletin.* Vol. XXVII, No. 1. New York: College Art Association of America.

Gailey, Alan. *Rural Houses of the North of Ireland.* Edinburgh: John Donald Publishers, Ltd. 1984.

Gailey reviews briefly the study of vernacular houses in Northern Ireland and then provides a detailed and well illustrated chapter on the early and medieval house. He then examines the houses in the study area in terms of structural systems, beginning with the social and logistical aspects of house size and construction, and then covers walls and wall materials, roofing and materials, hearth and chimney and flooring. The chapter on house types is well illustrated and presents a workable classification system accompanied by clear floor plans that represent the types, as well as excellent photographs and some distribution maps. The final chapters cover house and society, farmhouse and farmyard, and conservation and restoration. Floor plans are well detailed and collected together by type and to the same scale. Plans are indexed alphabetically in the general index making it easy to go to a specific plan.

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Glassie, Henry. "A Central Chimney Continental Log House." *Pennsylvania Folklife.* Lancaster: The Pennsylvania Folklife Society, Winter, 1968-1969.

Glassie locates the study area and the houses within the study both geographically and historically. He then sets out a structural grammar for the houses of the study area. Distribution maps present the distribution of house types within the study area, and the analysis of specific buildings is accompanied by large, careful drawings and accompanying photographs. The mechanics of innovation is examined and the final two chapters examine the social context of the house on the landscape and the historical context of the house in time.

________ *The Stars of Ballymenone*. Bloomington: Indiana Un iversity Press, 2006.

________ *Folk Housing in Middle Virgini.a* Knoxville: The University of Tennessee Press, 1975.

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Helm details the vernacular Frankish house and stable, profusely illustrated with paintings , art drawings and architectural drawings, both by Helm himself and period works dating to the fifteenth century including all pertinent works of Albrect Dürer. His analysis is geometrical in nature, based on the imposition of the square, the pentagon and the hexagon, or six-pointed star, on the elevation of the structure. He relates the use of the pentagon to folk beliefs about the *Drudendfuss*,or pentagram, and its ability, as a means to protect the household, to entrap the devil. It also contains an important discussion of some 3000 *Bauzeichnungen*, floor plans and elevations of which some date to the fifteenth century and archived before the war in Nürnberg, of which some few hundred are known to survive today.

________. Das *Bauernhaus im Gebiet der freien Reichstadt Nürnberg.* Berlin: Herbert Stubenrauch Verlagsbuchhandlubng, 1940.

Hewett, Cecil Alec. *The Development of Carpentry, 1200 – 1700.* Newton Abbot: David and Charles Publishers, 1969. Organized chronologically, presenting each period by means of specific examples of representative structures, and illustrated with plates of framing models and drawings of both whole constructed works and framing details. The descriptive text compares the details chronologically and developmentally.

Hiscock, Nigel. *The Wise Masterbuilder.* Brookfield: Ashgate Publishing Ltd., 2000.

James, John. *Chartres, The Masons who Built a Legend.* Boston: Routledge and Kegan Paul, 1982.
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Jenkins, J. Marshall. "Ground Rules of Welsh Houses: A Primary Analysis." *Journal of the Society for Folk Life Studies.* ed. J. Geraint Jenkins. Vol. 5, 1967.
This is the most detailed study I know of on the geometric use of square root of two relationships in vernacular architecture. Based on origins theory, it examines the relationship of the elements of the circle, radius, diameter and hypotenuse, as they are expressed in the primitive conical and walled circular hut. He applies these principles to many of the Welsh floor plans in Peate's *Welsh Houses.*

He is successful in demonstrating the presence of these relationships, but basing his work on rectangles expanded from circles of varying size and their associated squares, he does not find the means to unify the various parts of the plan. He reduces all geometric relationships to arithmetic expressions and these are often very difficult to follow. His plans and diagrams are much more effective in communicating the principles he tries to explain. The article is successful in demonstrating the use of the relationships, but fails to bring the observations together into an effective system.

Johnson, Matthew. *Housing Culture.* Washington D.C.: Smithsonian Institution Press, 1993. The book opens with a theoretical treatment of vernacular architecture, a geographic introduction to the Suffolk area, a study of meaning, and the development of a structural grammar for vernacular architecture. He divides the 116 houses of the study into open, transitional and closed houses, broadly corresponding to Gailey's direct entry and hearth-lobby entry. The floor plans are carefully drawn and the plates contain compass orientation, scale and the outline of significant details of decoration. He relates the closed house socially to status by class, describing in detail the associated process of land enclosure, and closes with a contextual analysis. The appendix lists all houses examined or whose records were consulted, and all probate inventories consulted in the course of research.

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